

Department of Aerospace Engineering, Indian Institute of Technology, Madras.

AS 5950: Continuum mechanics. Quiz 2

C slot, 22 March 2017

The grader is incapable of mind-reading. Write down all arguments and steps .

1. (10 marks) Let $\mathbf{C}$, $\mathbf{F}$ and $\mathbf{D}$ have the meanings assigned in class. Show that

$$\frac{D\mathbf{C}}{Dt} = 2\mathbf{F}^T \cdot \mathbf{D} \cdot \mathbf{F}. \quad (1)$$

2. (10 marks) Recall that $\mathbf{W}$ denotes the spin or vorticity tensor, and that $\mathbf{v}$ denotes the velocity. Show that

$$\frac{D\mathbf{v}}{Dt} = \frac{\partial \mathbf{v}}{\partial t} + \frac{\nabla(\mathbf{v} \cdot \mathbf{v})}{2} + 2\mathbf{W} \cdot \mathbf{v}, \quad (2)$$

where ∇ denotes the spatial gradient.

3. (10 marks) Consider a singular surface propagating with a certain speed normal to itself. The motion of the body is described by $\mathbf{x} = \mathbf{x}(\mathbf{X}, t)$ so that $\mathbf{F} = \partial \mathbf{x} / \partial \mathbf{X}$. Let $\mathbf{l}$ be any spatial vector instantaneously contained in the surface. Show that $\mathbf{F} \cdot \mathbf{l}$ is continuous across the singular surface.
4. (10 marks) Prove that W is positive for an isotropic linear elastic solid if $\mu > 0$, and $\lambda + 2\mu/3 > 0$.