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AS 5950: Continuum mechanics. Quiz 1

C slot, 15 February 2017

The grader is incapable of mind-reading. Write down all arguments and steps.

1. (**10 marks**) Let $\boldsymbol{v}$ be a vector field and ϕ be a scalar field. Using indicial notation prove that

$$\nabla \cdot (\phi \boldsymbol{v}) = \phi \nabla \cdot \boldsymbol{v} + \boldsymbol{v} \cdot \nabla \phi. \tag{1}$$

2. (**10 marks**) Let $\boldsymbol{Q}$ be an orthnormal tensor, and let $\boldsymbol{e}$ be a vector such that $\boldsymbol{Q} \cdot \boldsymbol{e} = \boldsymbol{e}$. Show that $\boldsymbol{Q}^T \cdot \boldsymbol{e} = \boldsymbol{e}$. What does this mean physically?

3. (**10 marks**) A motion is called simple shear if the velocity field has the form

$$\boldsymbol{v}(\boldsymbol{x}, t) = v_1(x_2)\boldsymbol{e}_1, \tag{2}$$

   in some cartesian frame. Show that in simple shear

$$\frac{Dv_i}{Dt}(x_j, t) = \frac{\partial v_i}{\partial t}(x_j, t). \tag{3}$$

4. (**20 marks**) Consider the motion

$$\begin{aligned} x_1 &= X_1 e^{t^2}, \\ x_2 &= X_2 e^{t}, \\ x_3 &= X_3, \end{aligned}$$

   where x_i denote spatial coordinates, X_R denote reference coordinates, and t denotes time.

   (a) Find the spatial description of velocity.

   (b) Find the components of the deformation gradient $F_{iR}(X_S, t)$.

   (c) Find the streamlines.