# Research Practice, Vibrations, Nonlinearities, Life :)

## Seminar to Structures Group, IIST Thiruvananthapuram

**Nidish Narayanaa Balaji**

**Indian Institute of Technology Madras, Chennai 600036, IN**

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# 1.1. My Journey So Far

Academia as a Career: Reflections

## Brief Bio

- BTech Aerospace (2017), **IIST**
- MS (2019) & PhD (2021) Mechanical, **Rice University**, Houston (adv: Dr. Matthew Brake) + Postdoc (2021-22)
- Humboldt Postdoctoral Researcher at **University of Stuttgart**, Stuttgart, DE (2022-24)

# 1.2. Reflections on Life in Academia

Academia as a Career: Reflections

- Life as a PhD student, as a Postdoc, as a faculty member

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- Academia in general

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Academia as a Career: Reflections

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- **Time is Fragmented**
- Academia in general
- Academia in India, US, Europe

# 1.3. Opportunities

Academia as a Career: Reflections

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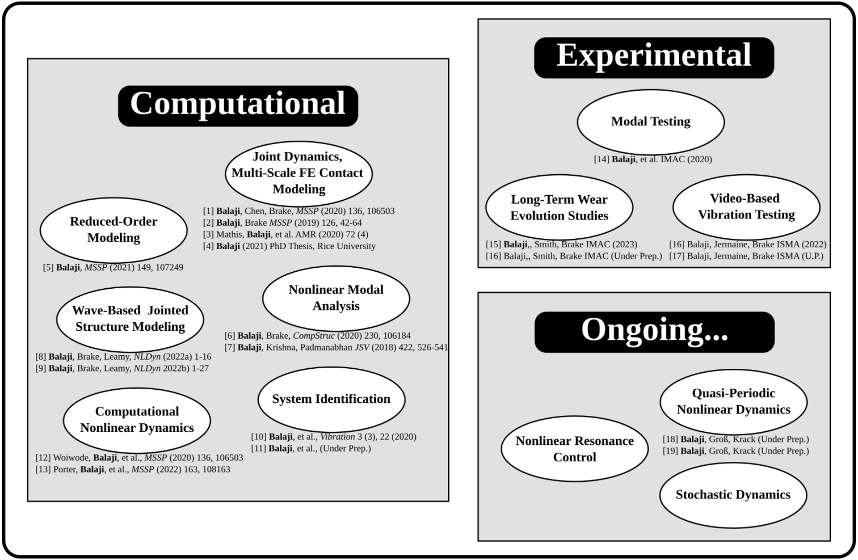
- Key is to ork on skills right now. **PhDs in engineering is not (yet) saturated.**
- **Most IIT's are mandated to grow. New IIT's coming up. Engineering faculty positions are not beyond your reach!**
- **Research-wise**, Indian academia is quite comfortable.

# 2. Highlights from my Research



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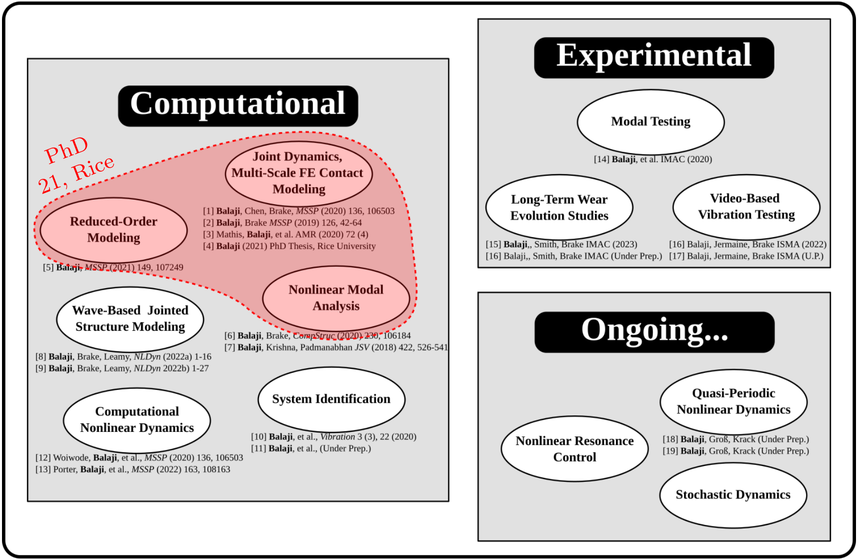

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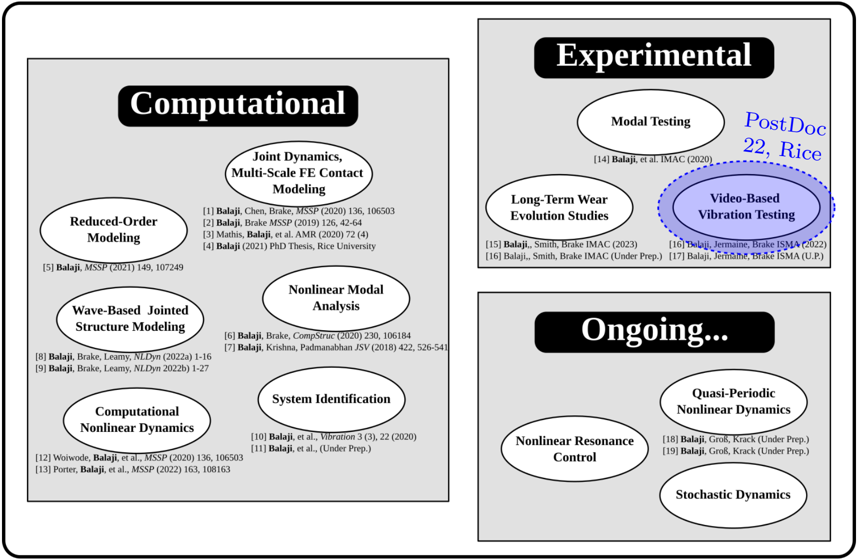

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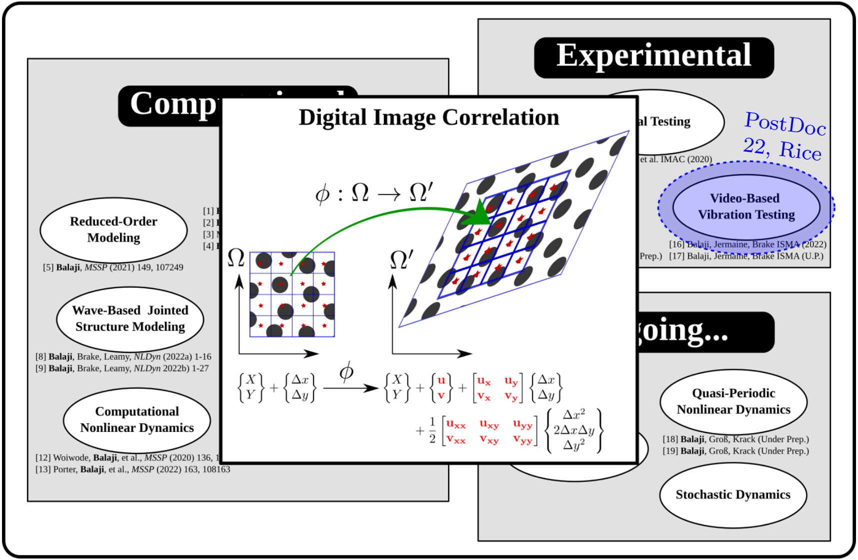

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**Joint Dynamics, Multi-Scale FE Contact Modeling**

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**Reduced-Order Modeling**

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**Wave-Based Jointed Structure Modeling**

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Collab. 22, GATech

**Nonlinear Modal Analysis**

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**Computational Nonlinear Dynamics**

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## Experimental

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## Ongoing...

**Quasi-Periodic Nonlinear Dynamics**

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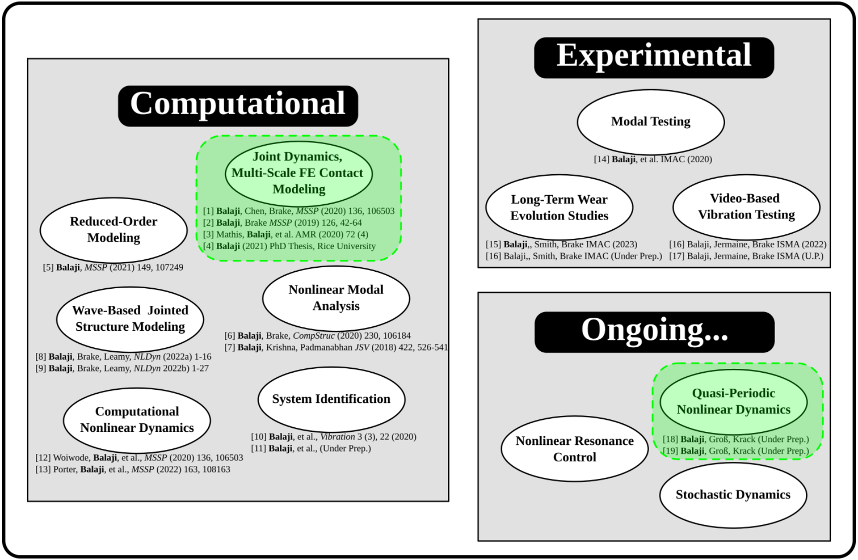

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## 3.1. Physics-Based Modeling of Bolted Joints

The Gaul Resonator (Süß, Janeba, and Willner 2018)

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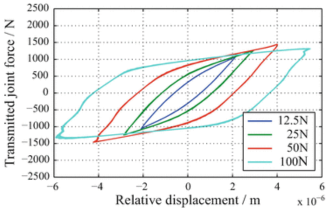

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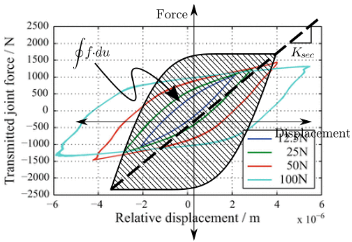

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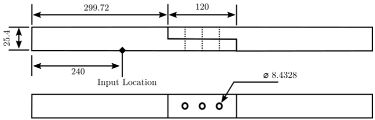
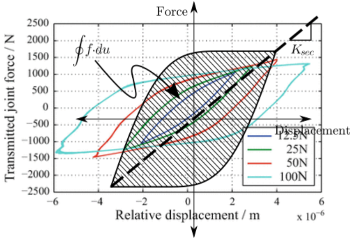



The Brake-Reuß Beam (Brake and Reuß 2018)



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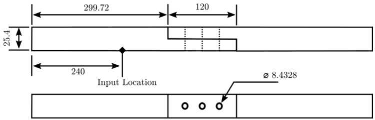


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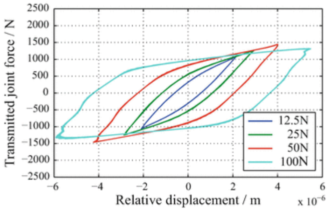
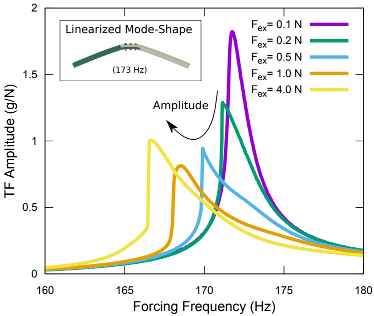


# 3. Nonlinear Dynamics of Jointed Structures

3.1. Physics-Based Modeling of Bolted Joints: Nonlinear Modal Analysis

## Rayleigh Quotient-based NMA

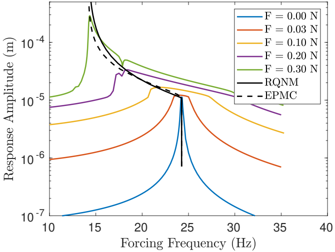
- The NMA is posed as an eigenvector-dependent non-linear Eigenvalue Problem (NEPv)

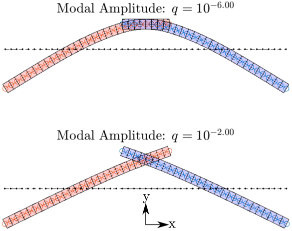
$$\underline{\underline{K}}\,\underline{u} + \underline{f_{nl}}(\underline{u}, \dots) - \underline{f_s} - \lambda \mathbf{M}(\underline{u} - \underline{u_s}) = 0$$

$$(\underline{u} - \underline{u_s})^T \mathbf{M}(\underline{u} - \underline{u_s}) - q^2 = 0.$$

- See (Balaji and Brake 2020) for details

## Example: Frictional Beam



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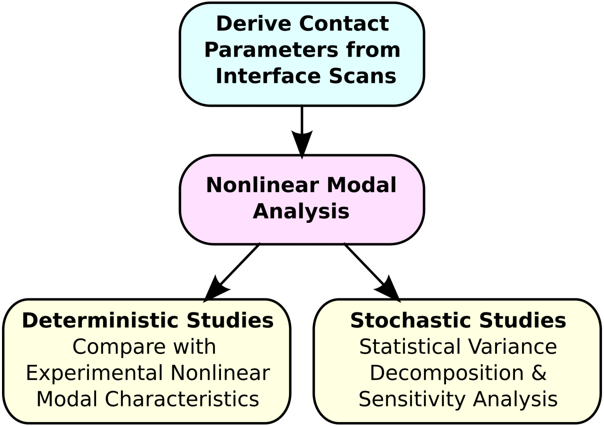


***The TriboMechaDynamics Approach***

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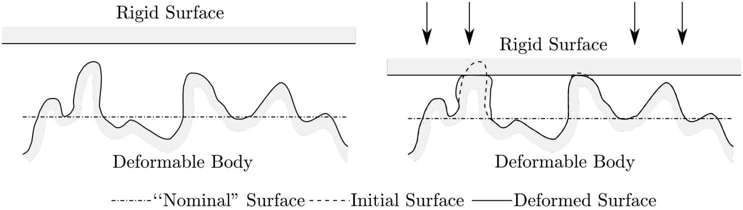


- The statistical treatment of rough contact has been popular in the contact mechanics community from (Greenwood and Williamson 1966)
- The idea is to describe the reaction force as a statistical expectation of asperity-reaction forces randomly distributed over a given surface

**Exponentially Distributed Surface**

- The asperity heights are fitted to a two parameter exponential distribution, following (Polycarpou and Etsion 1999; Medina, Nowell, and Dini 2013)

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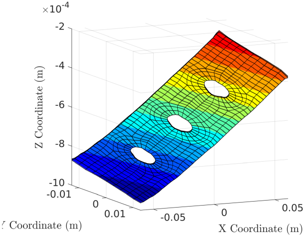
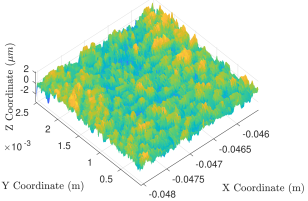
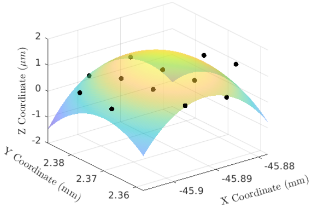
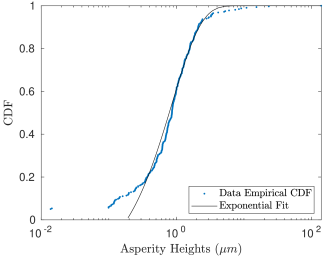
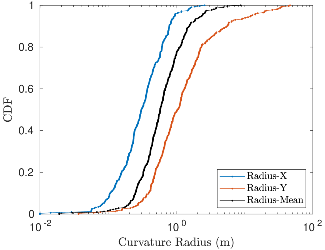
## 3.1. Physics-Based Modeling of Bolted Joints: Contact Parameter Estimation



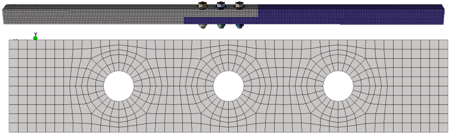



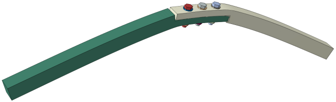



# 3. Nonlinear Dynamics of Jointed Structures

## 3.1. Physics-Based Modeling of Bolted Joints: Modeling Methodology

**Linear finite element model with only contact non-linearities**





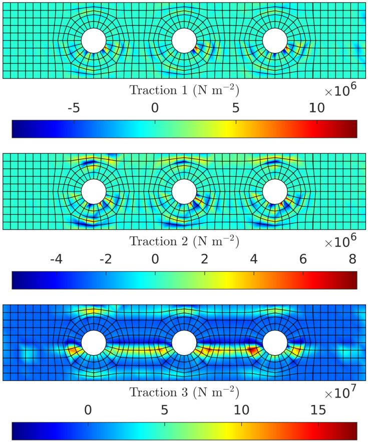
***Factors for Uncertainty Propagation***

| S.No. | Description | Symbol | Distribution | Quadrature |
|---|---|---|---|---|
| 1. | Coefficient of Friction | $\mu$ | Exponential (mean$\approx$ 0.1183) | Gauss-Laguerre |
| 2. | Gap Function | $g$ | Normal (fit parameters) | Gauss-Hermite |
| 3. | Asperity height exp. | $\lambda$ | Normal (fit parameters) | Gauss-Hermite |
| 4. | Mean Radius | $R$ | Normal (fit parameters) | Gauss-Hermite |
| 5. | Stage Rotation X | $\theta_X$ | Normal (0 mean, 15° s.d.) | Gauss-Hermite |
| 6. | Stage Rotation Y | $\theta_Y$ | Normal (0 mean, 15° s.d.) | Gauss-Hermite |
| 7. | Bolt Prestress Force | $P$ | Normal (exp. mean, s.d.) | Gauss-Hermite |

# 3. Nonlinear Dynamics of Jointed Structures

## 3.1. Physics-Based Modeling of Bolted Joints: Mean Model Results

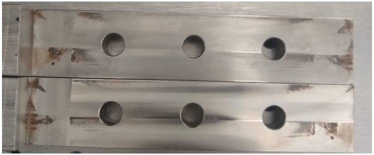
### Static Tractions




### Prediction of Linearized Natural Frequency

| S.No. | Exp. (Hz) | Mean Model (Hz) | Error (%) |
|---|---|---|---|
| 1 | 179.56 | 179.41 | **0.0845** |
| 2 | 594.71 | 594.72 | **0.0016** |
| 3 | 1199.8 | 1197.1 | **0.2209** |

### The interfaces after several hours of testing



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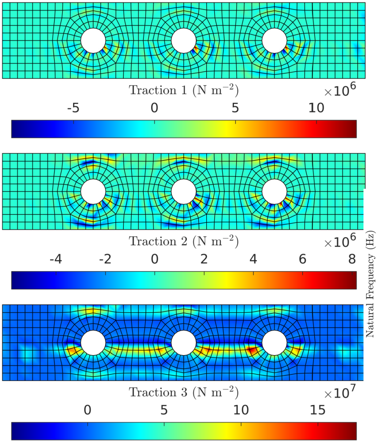


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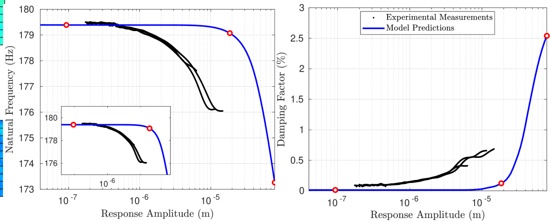
### Mode 1 Dynamics



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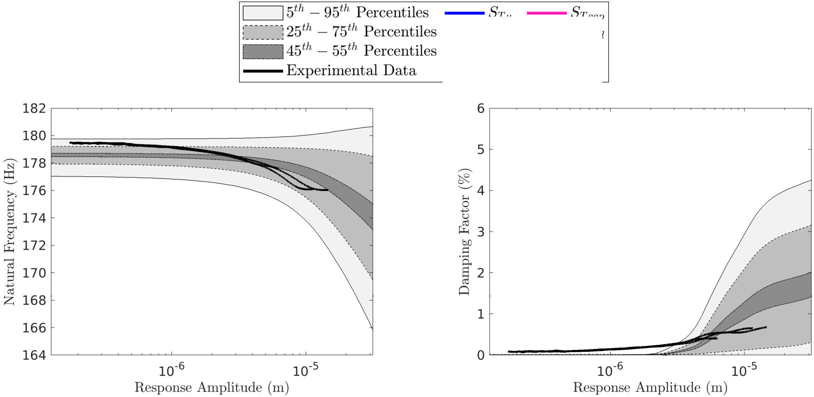

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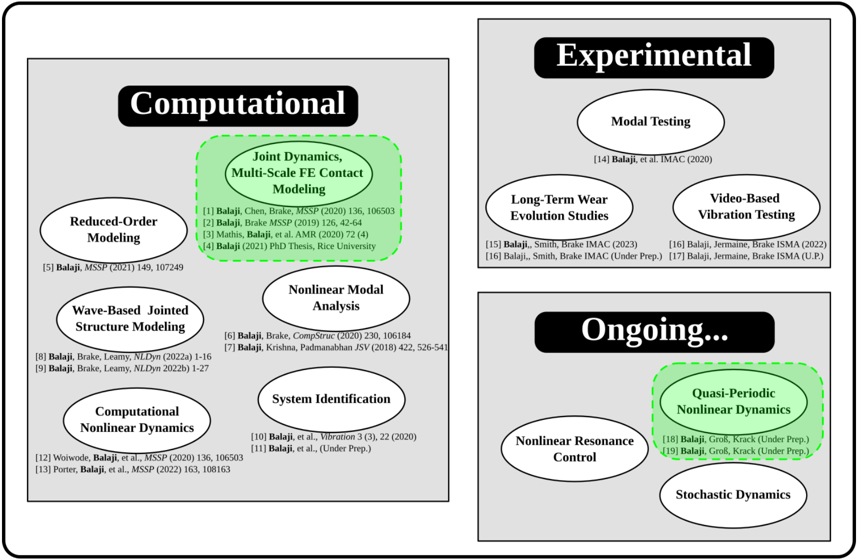

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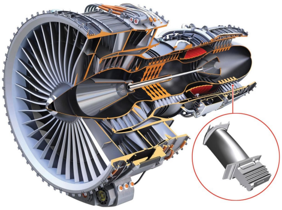
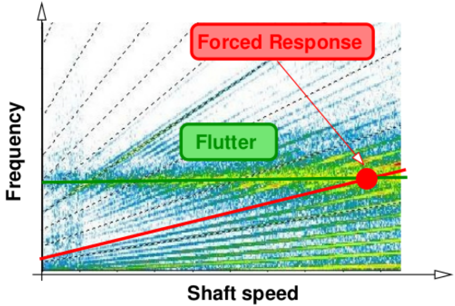
## 3.2. Self-Excited Oscillations

- Multiple sources of excitation in jet-engines






*Blade stage vibration data from MTU aero test engines (Corral, Gallardo, and Ivaturi 2013)*

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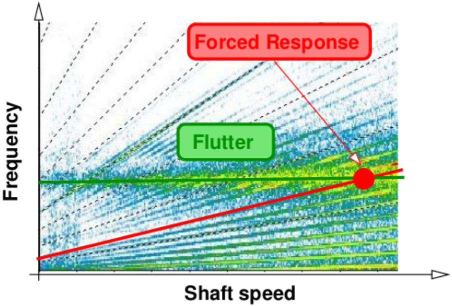
## 3.2. Self-Excited Oscillations

- Multiple sources of excitation in jet-engines
- The aerodynamic interactions can sometimes be modeled as a **self-excitation** for a traveling wave mode

$$\ddot{\eta} + c\dot{\eta} + \omega_0^2\eta = 2\omega_0\zeta\dot{\eta}$$
$$\implies \ddot{\eta} + (c - 2\omega_0\zeta)\dot{\eta} + \omega_0^2\eta = 0$$

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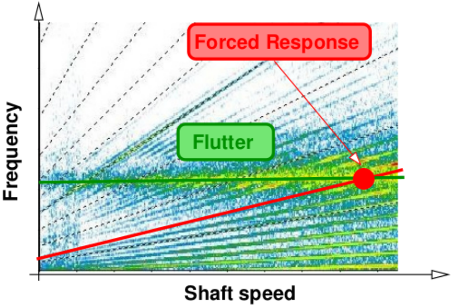
- These structures are often supported by frictional contacts and the "modal" equations become

$$\ddot{\eta} + (c - 2\omega_0\zeta)\dot{\eta} + \omega_0^2\eta + f_{nl}(\eta, \dots) = 0.$$

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# 3. Nonlinear Dynamics of Jointed Structures

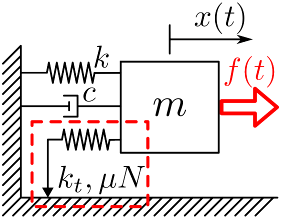
## 3.2. Self-Excited Oscillations: A Self-Excited SDOF Benchmark

- A representative problem is analyzed

$$m\ddot{x} - c\dot{x} + kx + f_{nl}(x,\dots) = \frac{F}{2}e^{j\Omega t} + c.c.$$

using a Harmonic Balance approach, with the Fourier ansatz:

$$x(t) = \sum_{k\in\mathcal{H}} U_k e^{jk\Omega t} + c.c.$$




- Quasi-Periodic HB Ansatz ($\tau_i = \Omega_i t$):

$$x(t) = \sum_{\underline{k}\in\mathcal{H}} U_{\underline{k}} \exp(i(k_1\tau_1 + k_2\tau_2))$$

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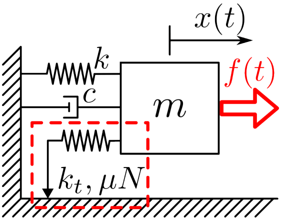


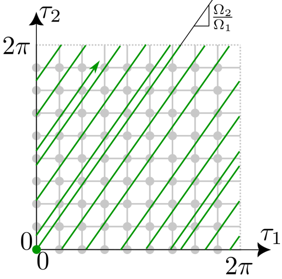

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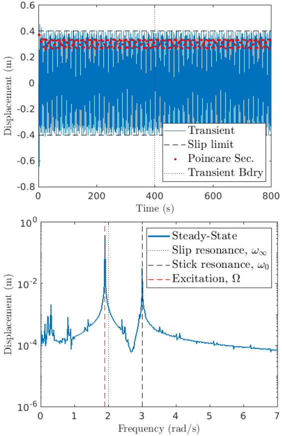
**Quasiperiodic**




$\Omega < \omega_{res}$

# 3. Nonlinear Dynamics of Jointed Structures

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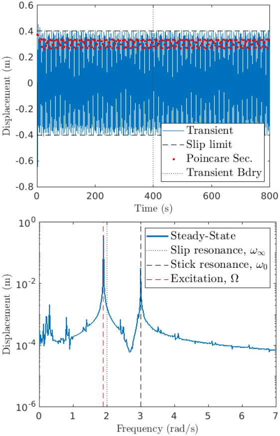
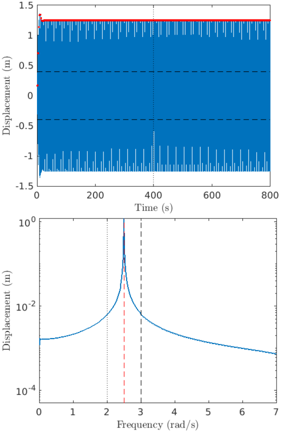



$\Omega < \omega_{res}$

**Periodic, Lock-in!**




$\Omega \sim \omega_{res}$

# 3. Nonlinear Dynamics of Jointed Structures

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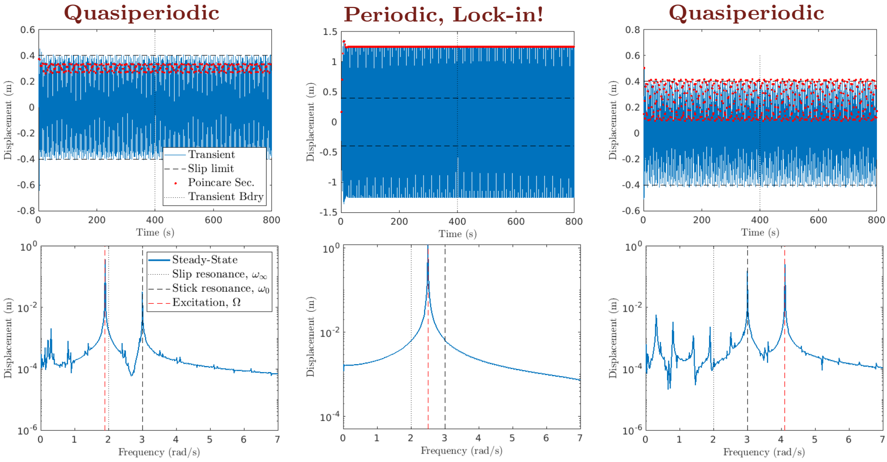
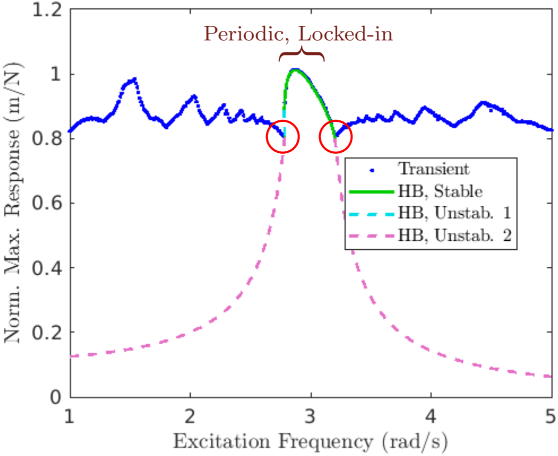



$\Omega < \omega_{res}$ $\Omega \sim \omega_{res}$ $\Omega > \omega_{res}$

# 3. Nonlinear Dynamics of Jointed Structures

## 3.2. Self-Excited Oscillations: A Self-Excited SDOF Benchmark



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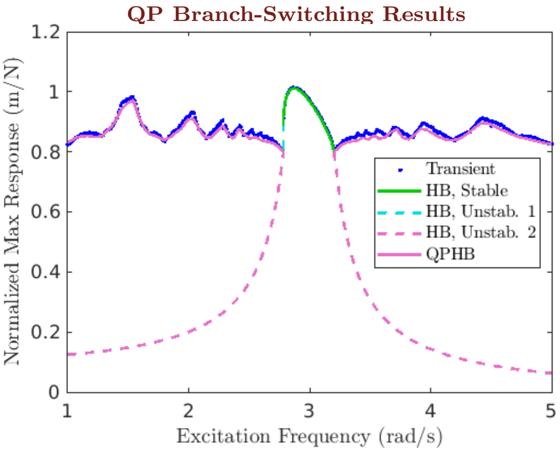
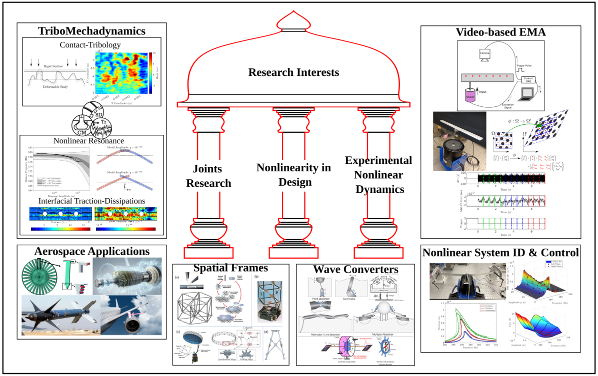


# 3. Nonlinear Dynamics of Jointed Structures

## 3.3. Research Plan Overview



# 3. Nonlinear Dynamics of Jointed Structures

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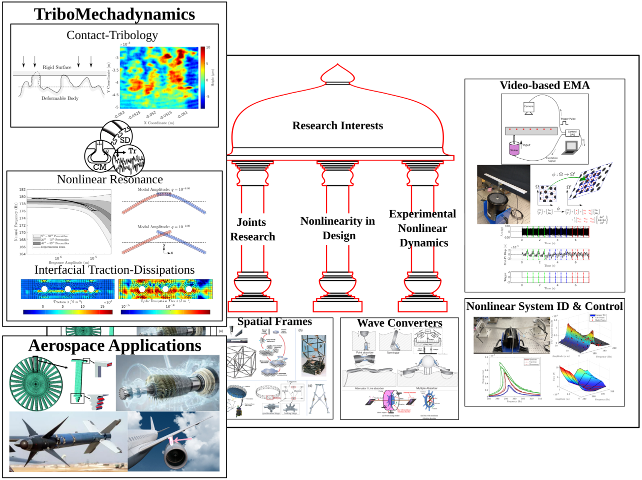

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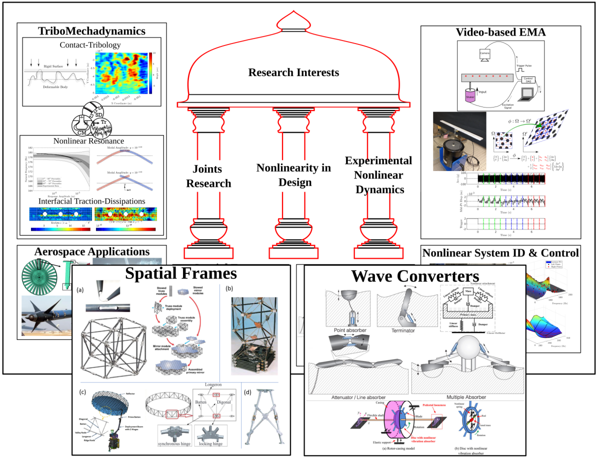
## 3.3. Research Plan Overview



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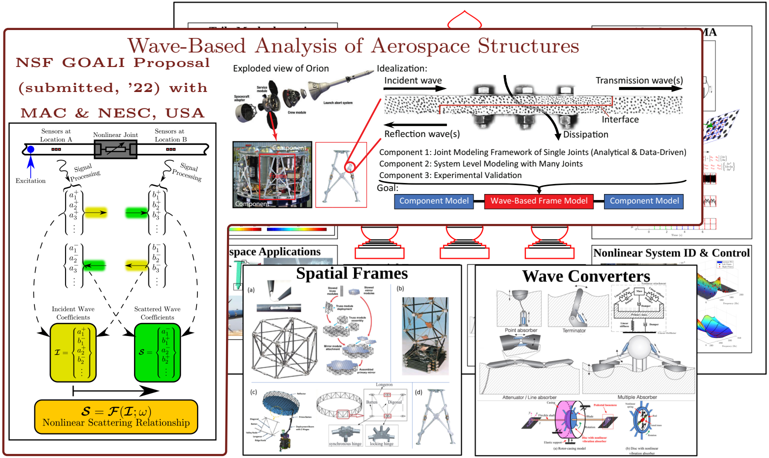

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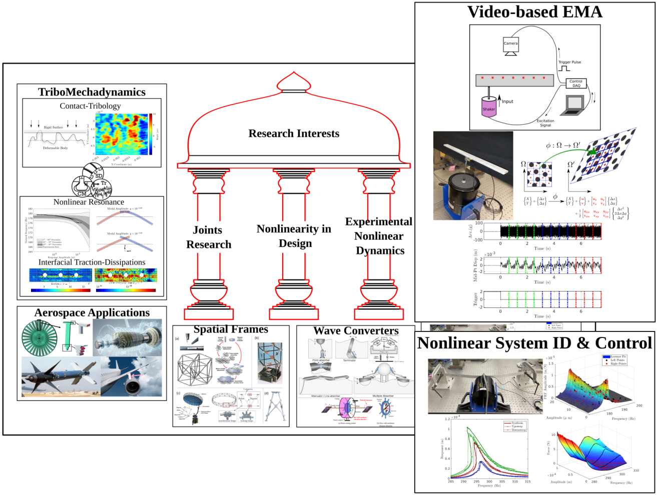
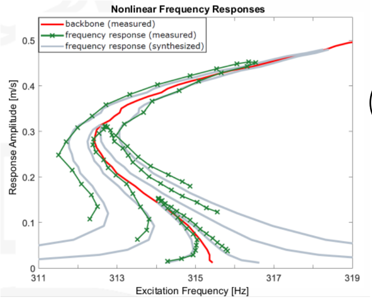
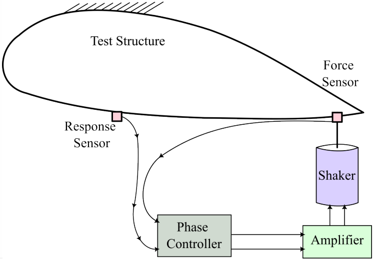


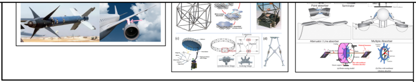
# 3. Nonlinear Dynamics of Jointed Structures

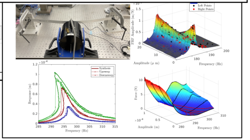
## 3.3. Research Plan Overview

### Phase-based Resonance Control





# 3. Nonlinear Dynamics of Jointed Structures

## 3.4. Focus on Free and Open Source Software

- **Big proponent of Free and Open Source Computing!**

# Computational

**Joint Dynamics, Multi-Scale FE Contact Modeling**

[1] **Balaji**, Chen, Brake, *MSSP* (2020) 136, 106503
[2] **Balaji**, Brake *MSSP* (2019) 126, 42-64
[3] Mathis, **Balaji**, et al. AMR (2020) 72 (4)
[4] **Balaji** (2021) PhD Thesis, Rice University

**Reduced-Order Modeling**

[5] **Balaji**, *MSSP* (2021) 149, 107249

**Nonlinear Modal Analysis**

[6] **Balaji**, Brake, *CompStruc* (2020) 230, 106184
[7] **Balaji**, Krishna, Padmanabhan *JSV* (2018) 422, 526-541

**Wave-Based Jointed Structure Modeling**

[8] **Balaji**, Brake, Leamy, *NLDyn* (2022a) 1-16
[9] **Balaji**, Brake, Leamy, *NLDyn* 2022b) 1-27

**System Identification**

[10] **Balaji**, et al., *Vibration* 3 (3), 22 (2020)
[11] **Balaji**, et al., (Under Prep.)

**Computational Nonlinear Dynamics**

[12] Woiwode, **Balaji**, et al., *MSSP* (2020) 136, 106503

# Experimental

**Modal Testing**

[14] **Balaji**, et al. IMAC (2020)

**Long-Term Wear Evolution Studies**

[15] **Balaji**,, Smith, Brake IMAC (2023)
[16] Balaji,, Smith, Brake IMAC (Under Prep.)

**Video-Based Vibration Testing**

[16] Balaji, Jermaine, Brake ISMA (2022)
[17] Balaji, Jermaine, Brake ISMA (U.P.)

# Ongoing...

**Quasi-Periodic Nonlinear Dynamics**

[18] **Balaji**, Groß, Krack (Under Prep.)
[19] **Balaji**, Groß, Krack (Under Prep.)

**Nonlinear Resonance Control**

**Stochastic Dynamics**

http://ae.iitm.ac.in/ nidish

*Thank You!*

*nidish@iitm.ac.in*

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D. Süß, A. Janeba, and K. Willner. "The Gaul Resonator: Experiments for the Isolated Investigation of a Bolted Lap Joint". In: *The Mechanics of Jointed Structures: Recent Research and Open Challenges for Developing Predictive Models for Structural Dynamics*. Ed. by M. R. Brake. Cham: Springer International Publishing, 2018, pp. 59–72. ISBN: 978-3-319-56818-8. DOI: 10.1007/978-3-319-56818-8_6. URL: https://doi.org/10.1007/978-3-319-56818-8_6 (cit. on pp. 20–24).

M. R. W. Brake and P. Reuß. "The Brake-Reuß Beams: A System Designed for the Measurements and Modeling of Variability and Repeatability of Jointed Structures with Frictional Interfaces". In: *The Mechanics of Jointed Structures: Recent Research and Open Challenges for Developing Predictive Models for Structural Dynamics*. Ed. by M. R. Brake. Cham: Springer International Publishing, 2018, pp. 99–107. ISBN: 978-3-319-56818-8. DOI: 10.1007/978-3-319-56818-8_9. URL: https://doi.org/10.1007/978-3-319-56818-8_9 (cit. on pp. 20–24).

N. N. Balaji and M. R. Brake. "A Quasi-Static Non-Linear Modal Analysis Procedure Extending Rayleigh Quotient Stationarity for Non-Conservative Dynamical Systems". Computers & Structures, **230**, (Apr. 2020), pp. 106184. ISSN: 00457949. DOI: 10.1016/j.compstruc.2019.106184. URL: https://linkinghub.elsevier.com/retrieve/pii/S0045794919315160 (cit. on p. 25).

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S. Medina, D. Nowell, and D. Dini. "Analytical and Numerical Models for Tangential Stiffness of Rough Elastic Contacts". Tribology Letters, **49**,1 (Jan. 2013), pp. 103–115. ISSN: 1023-8883, 1573-2711. DOI: 10.1007/s11249-012-0049-y. URL: http://link.springer.com/10.1007/s11249-012-0049-y (cit. on p. 27).

R. Corral, J. M. Gallardo, and R. Ivaturi. "Conceptual Analysis of the Non-Linear Forced Response of Aerodynamically Unstable Bladed-Discs". In: *Volume 7A: Structures and Dynamics*. San Antonio, Texas, USA: American Society of Mechanical Engineers, June 2013, pp. V07AT32A001. ISBN: 978-0-7918-5526-3. DOI: 10.1115/GT2013-94851. URL: https://asmedigitalcollection.asme.org/GT/proceedings/GT2013/55263/San%20Antonio,%20Texas,%20USA/244495 (cit. on pp. 34–36).

M. Jin et al. "Identification of Instantaneous Frequency and Damping From Transient Decay Data". Journal of Vibration and Acoustics, **142**,5 (June 2020). ISSN: 1048-9002. DOI: 10.1115/1.4047416. URL: https://doi.org/10.1115/1.4047416 (cit. on pp. 55–59).

N. Wiener. "The Homogeneous Chaos". American Journal of Mathematics, **60**,4 (1938), pp. 897–936. ISSN: 0002-9327. DOI: 10.2307/2371268. JSTOR: 2371268. URL: https://www.jstor.org/stable/2371268 (cit. on pp. 71–74).

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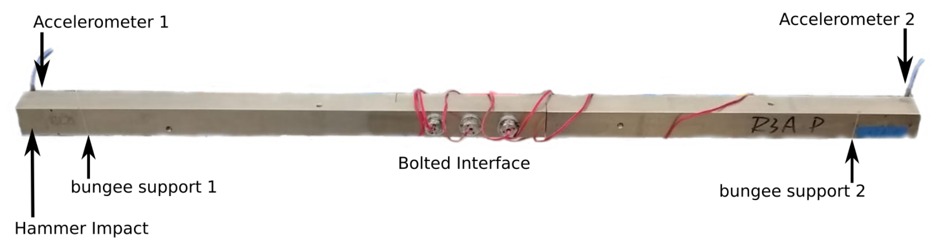
N. N. Balaji, J. Groß, and M. Krack. “Computational Approaches for Quasi-Periodic Hysteretic Dynamics”. (2023, Under Preparation) (cit. on pp. 77–85).

# 5. Experimental Details

## Hammer Impact Testing Setup



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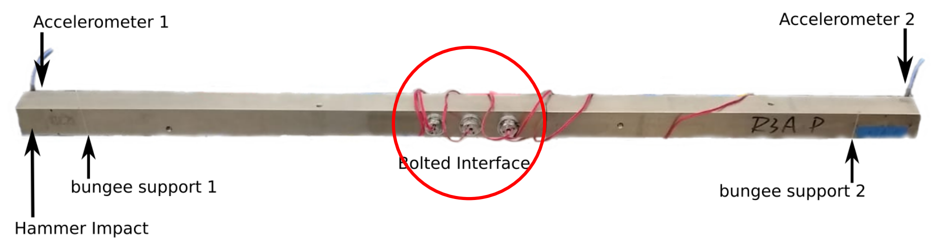
## Hammer Impact Testing Setup



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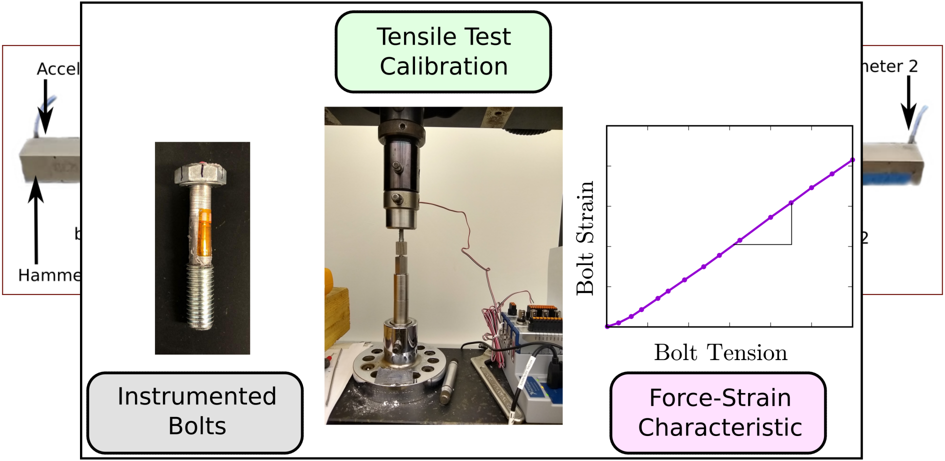

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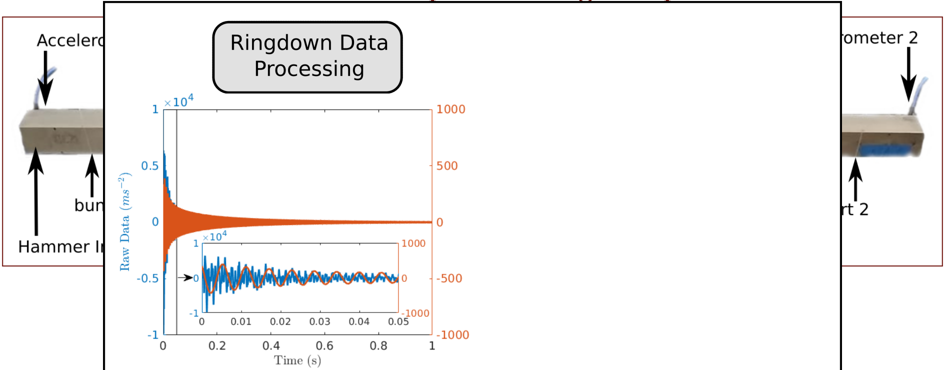


(Jin et al. 2020)

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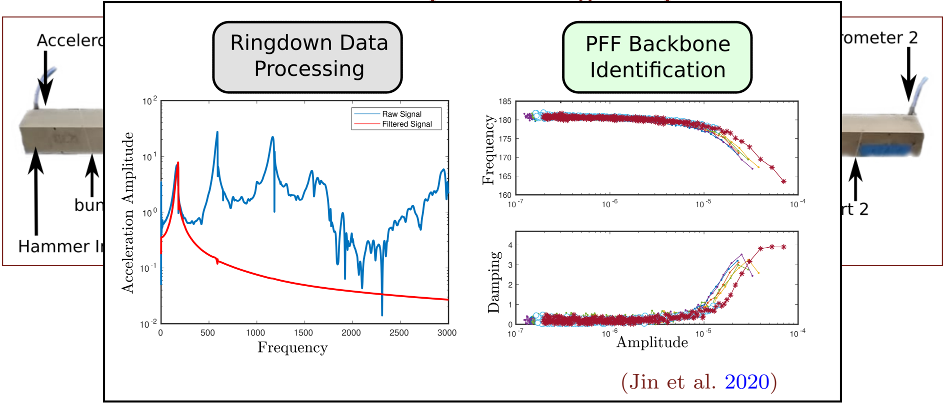


(Jin et al. 2020)

# 5. Experimental Details

## Interface Scanning and Processing


The Keyence VR-5100 White Light Interferometer

# 5. Experimental Details

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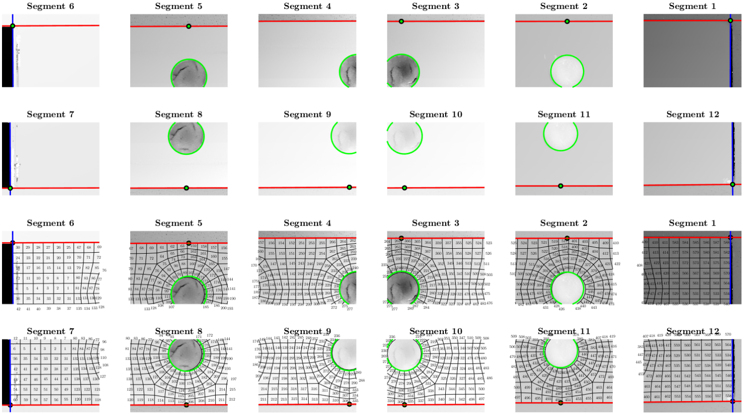

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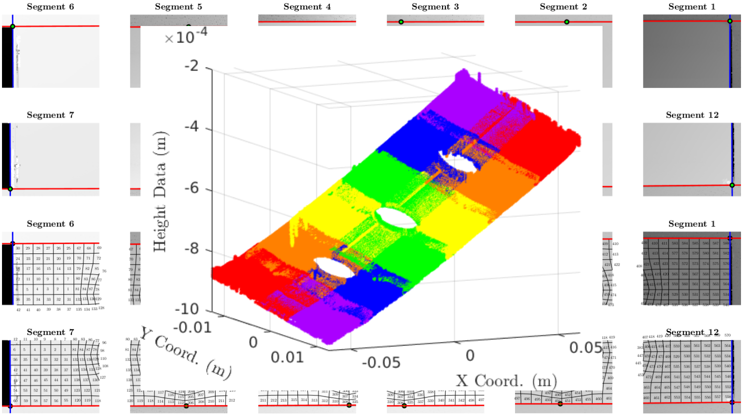

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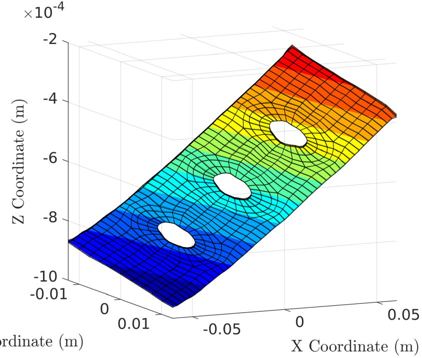
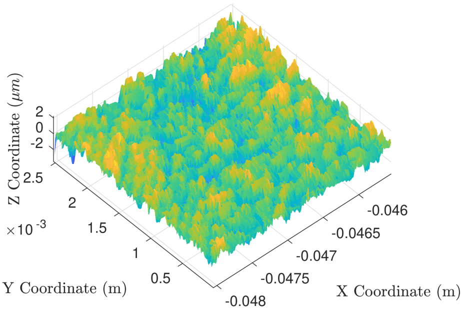


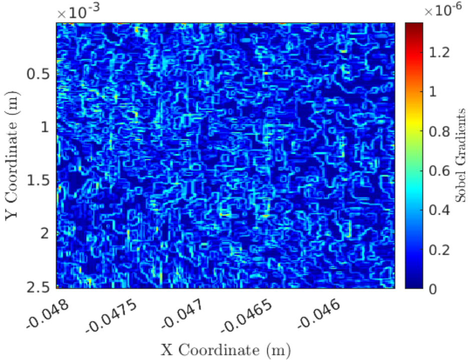

**Meso-scale Topography**



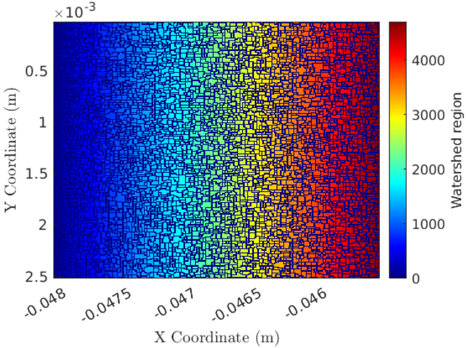

**Micro-scale Asperity Distribution**

# 5. Experimental Details

## Interface Scanning and Processing



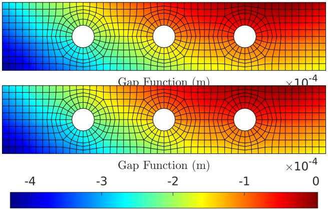

**Sobel Gradients**

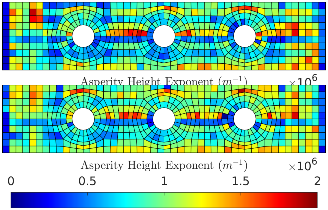


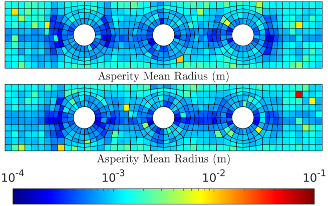

**Watershed Regions**

# 5. Experimental Details

## Contact Parameter Estimation



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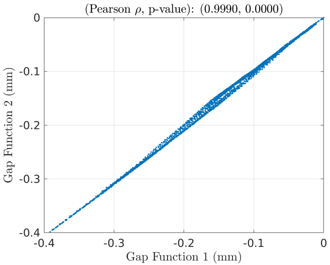
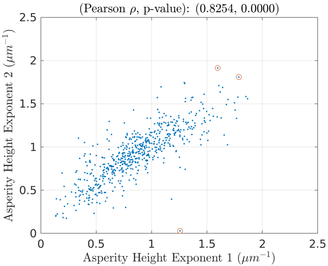
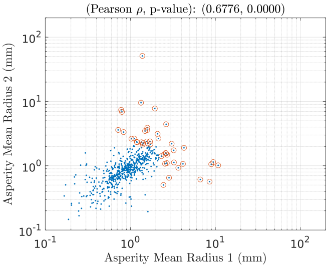
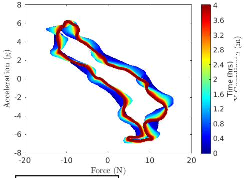


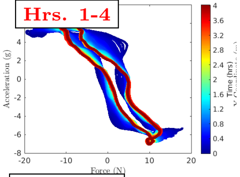



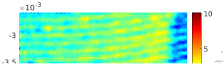
# 6. Wear Evolution

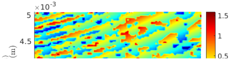


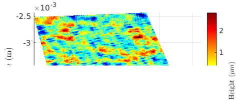

Hrs. 1-4

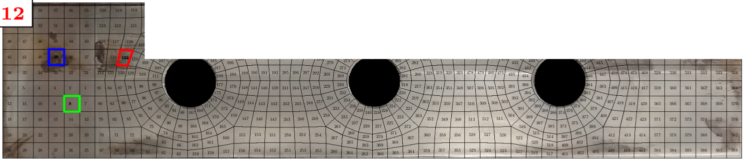



Hrs. 9-12



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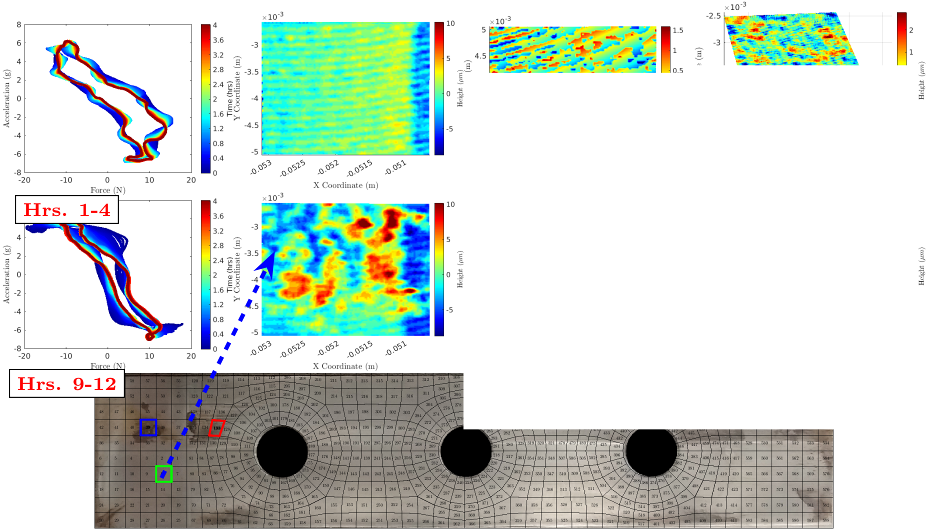

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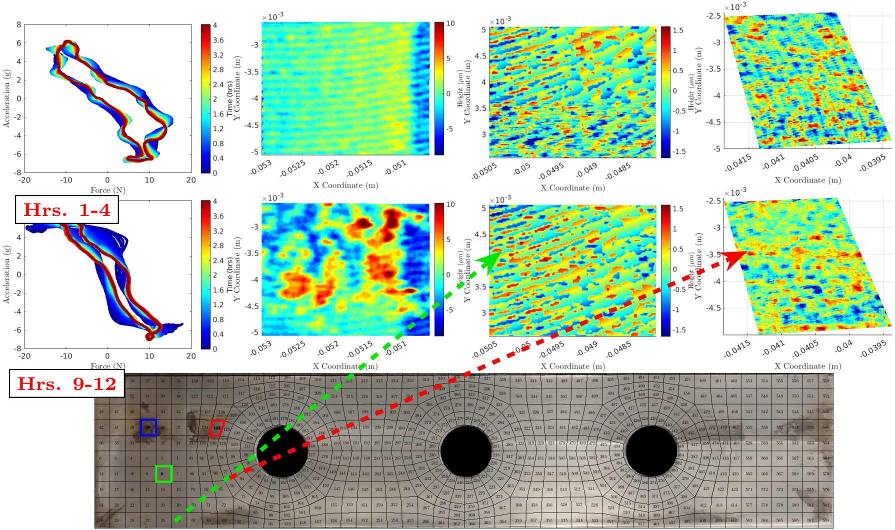

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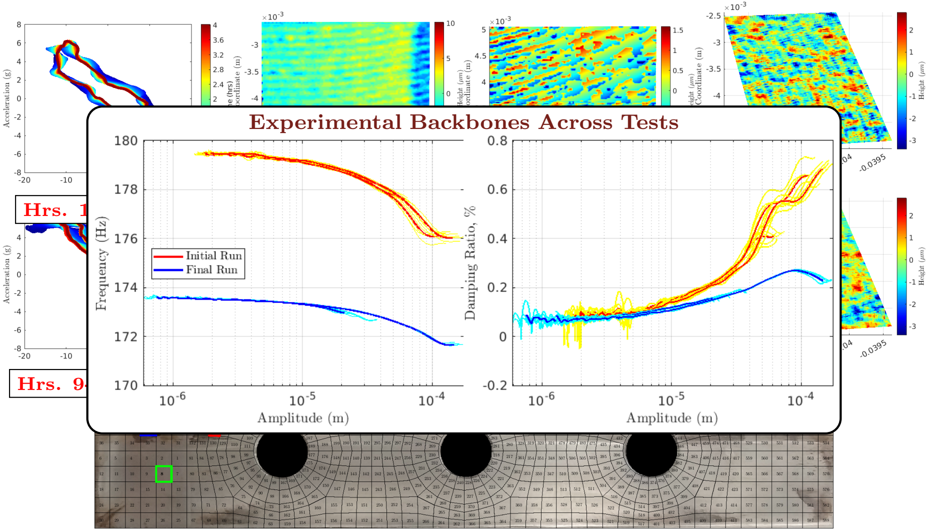

# 7. Stochastic Modeling through Polynomial Chaos Expansion

- The Polynomial Chaos Expansion (PCE) approach is adopted for the stochastic modeling purpose (Wiener 1938; Sudret 2008)

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$$y \leftarrow f(x) \qquad x \in \mathcal{D} \sim w(.) \qquad (\text{pdf: } w(x))$$

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- The family of polynomials $\psi_n(x)$ that are orthogonal with respect to the inner product weighted by $w(x)$ are chosen as the bases for the PCE

$$\langle \psi_n, \psi_m \rangle = \int_{\mathcal{D}} \psi_n(x) \psi_m(x) w(x) dx = \mathbb{E}[\psi_n \psi_m] = \delta_{mn}.$$

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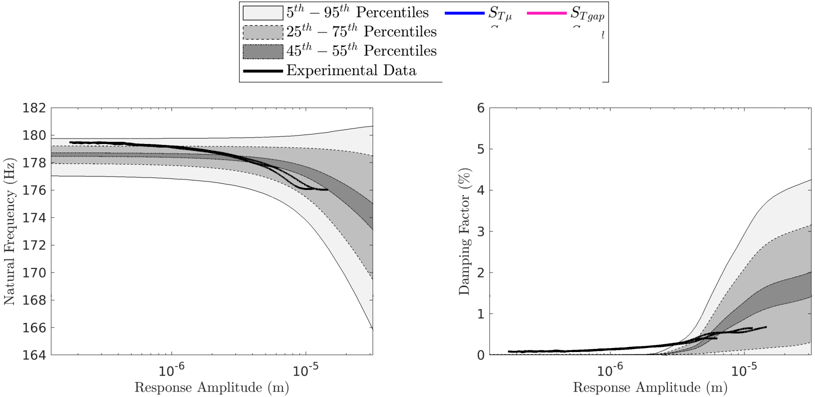
- The Polynomial Chaos Expansion of $y$ is written as

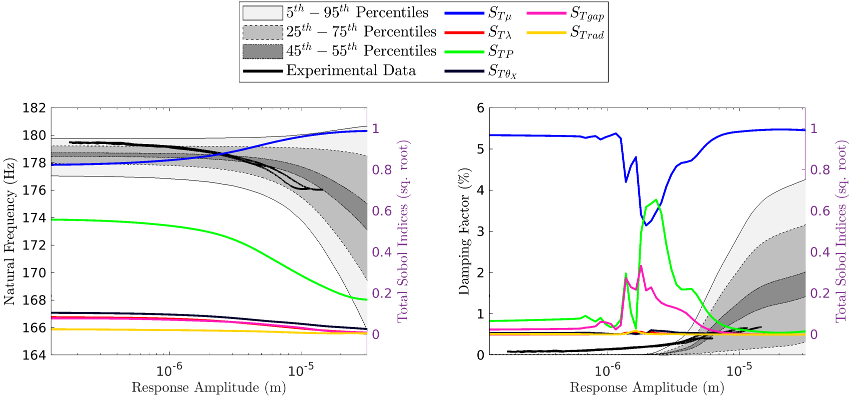
$$\hat{y} = f_0 + \sum_{i=1}^{N} f_i \psi_i(x) \rightarrow \hat{y} = f_0 + \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} f_{ij} \psi_i^{(1)}(x_1) \psi_j^{(2)}(x_2).$$

**Variance Decomposition**

# 8. PCE Results



# 8. PCE Results



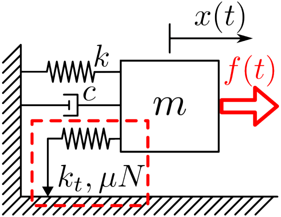
# 9. A Self-Excited SDOF Benchmark

- A representative problem is analyzed

$$m\ddot{x} - c\dot{x} + kx + f_{nl}(x, \dots) = \frac{F}{2} e^{j\Omega t} + c.c.$$

using a Harmonic Balance approach, with the Fourier ansatz:

$$x(t) = \sum_{k \in \mathcal{H}} U_k e^{jk\Omega t} + c.c.$$



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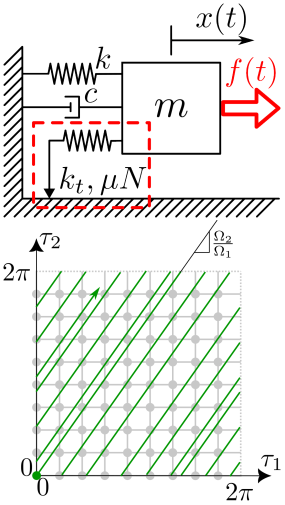
$$x(t) = \sum_{k \in \mathcal{H}} U_k e^{jk\Omega t} + c.c.$$

- Quasi-Periodic HB Ansatz ($\tau_i = \Omega_i t$):

$$x(t) = \sum_{\underline{k} \in \mathcal{H}} U_{\underline{k}} \exp(i(k_1 \tau_1 + k_2 \tau_2))$$



# 9. A Self-Excited SDOF Benchmark

- A representative problem is analyzed

$$m\ddot{x} - c\dot{x} + kx + f_{nl}(x, \dots) = \frac{F}{2}e^{j\Omega t} + c.c.$$

using a Harmonic Balance approach, with the Fourier ansatz:

$$x(t) = \sum_{k \in \mathcal{H}} U_k e^{jk\Omega t} + c.c.$$

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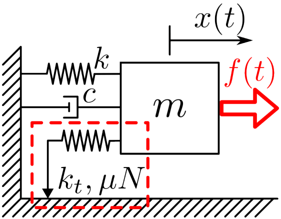
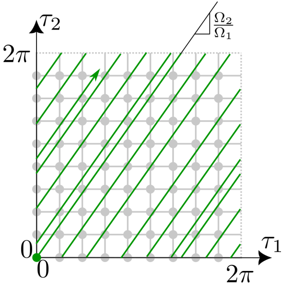
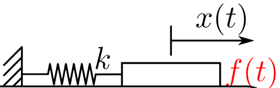



**Evaluation of Frictional Forces**

$$f_i = \begin{cases} \overbrace{k_t(x_i - x_{i-1}) + f_{i-1}}^{f^{(sp)}} & stick \\ \mu N sign(f^{(sp)}) & slip \end{cases}$$

# 9. A Self-Excited SDOF Benchmark

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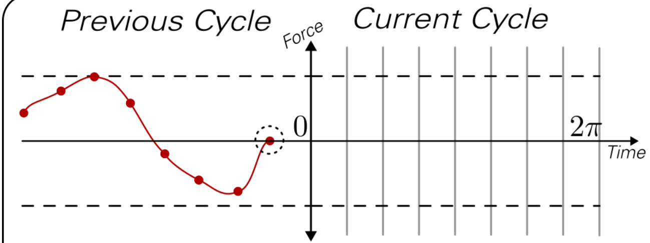


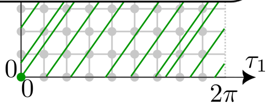


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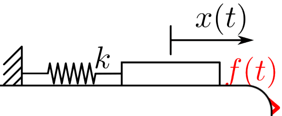


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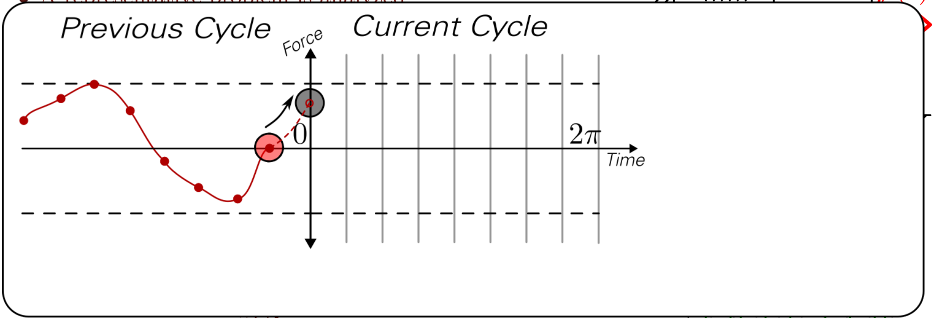


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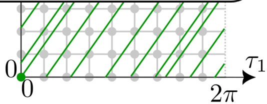

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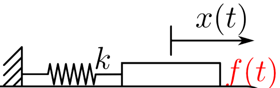


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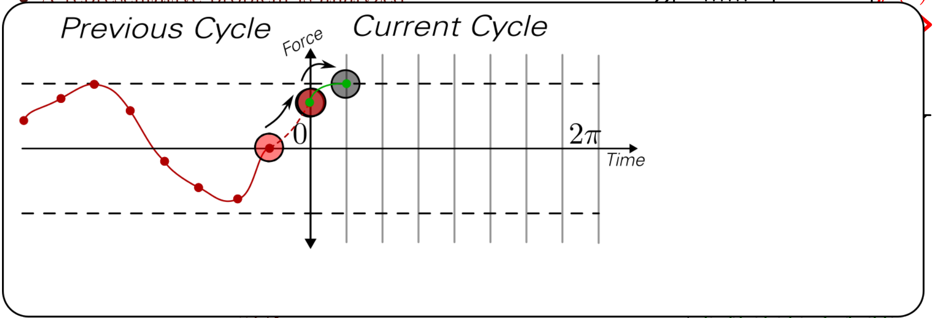


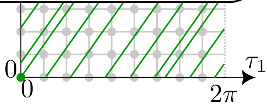


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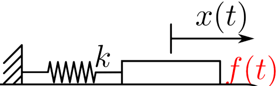


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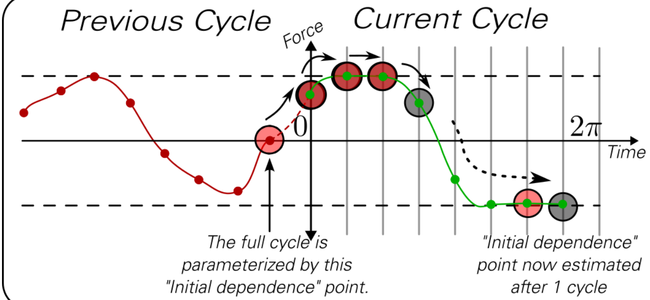


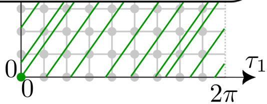


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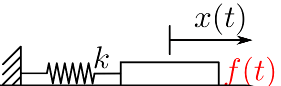


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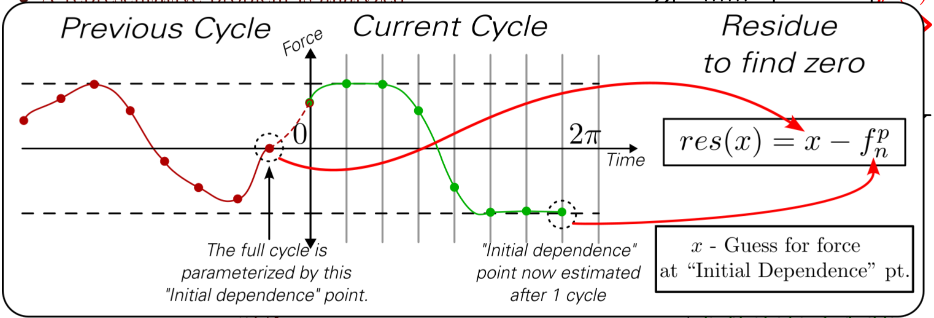


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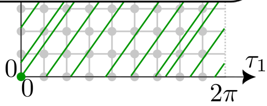

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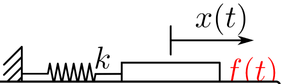


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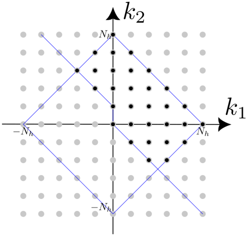
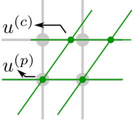
**QPHB Hyper-Time Marching (see Balaji, Groß, and Krack 2023, Under Preparation)**

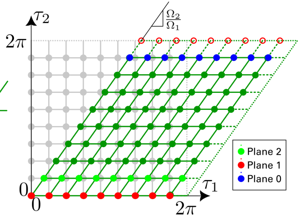
$$x(t) = \sum_{\underline{k} \in \mathcal{H}} U_{\underline{k}} \exp(i(k_1 \tau_1 + k_2 \tau_2))$$

**Harmonic Truncation**

$$\mathcal{H} = \{(k_1, k_2) | \quad [|k_1| + |k_2| \leq N_h] \cap [k_1 + k_2 > 0] \cap [(k_1 + k_2 = 0 \cap k_1 > 0)]\}$$



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