AS3020: Aerospace Structures

Module 4: Bending of Beam-Like Structures

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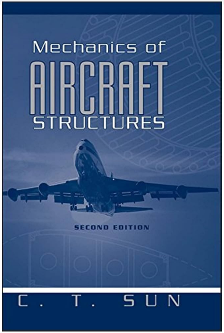
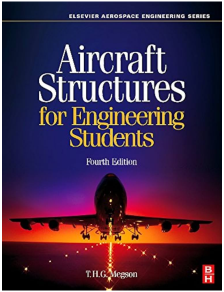
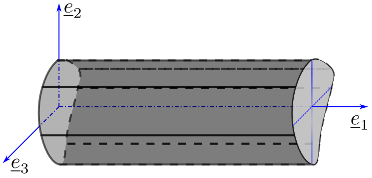


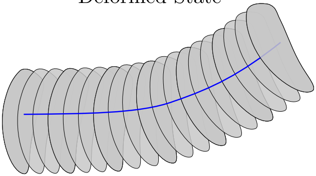
Chapters 4-5 in Sun [1]



Chapters 16-20 in Megson [2]

1. Unsymmetrical Bending





- Displacement Field

$$u_1 = -X_2\theta_3 + X_3\theta_2, \quad u_2 = v, \quad u_3 = w.$$

- Zero shear $\implies \theta_3 = v\prime, \quad \theta_2 = -w\prime$
- Direct stress

$$\sigma_{11} = E_y \begin{bmatrix} X_3 & -X_2 \end{bmatrix} \begin{bmatrix} \theta_2\prime \\ \theta_3\prime \end{bmatrix}$$
$$= \frac{\begin{bmatrix} X_3 & -X_2 \end{bmatrix}}{I_{22}I_{33} - I_{23}^2} \begin{bmatrix} I_{33}M_2 + I_{23}M_3 \\ I_{23}M_2 + I_{22}M_3 \end{bmatrix}$$

- Equilibrium Considerations:

$$M_{2,1} = V_3, \quad V_{2,1} + F_2 = 0$$
$$M_{3,1} = -V_2, \quad V_{3,1} + F_3 = 0.$$

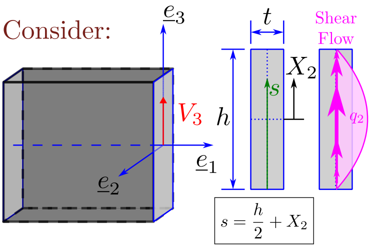
2. Shear Stress in Sections

- If shear strain is assumed zero, can we still have shear stress?
- We posit: $\gamma_{12} = 0$, $\gamma_{13} = 0$, $\gamma_{23} = 0$. As point quantities, the shear stresses may still be small ($\tau_{12} = G\sigma_{12}$).
- But the **integral quantities** are taken to be finite:

$$\int \sigma_{12} dA = V_2, \qquad \int \sigma_{13} dA = V_3,$$

$$\int \sigma_{12} dX_3 = q_2, \qquad \int \sigma_{13} dX_2 = q_3.$$

- Consider:



$$q_3(s) = \frac{V_3}{I_{22}} \int_0^s t X_2 ds = \frac{tV_3}{I_{22}} \int_{-\frac{h}{2}}^{X_2} X_2 dX_2$$

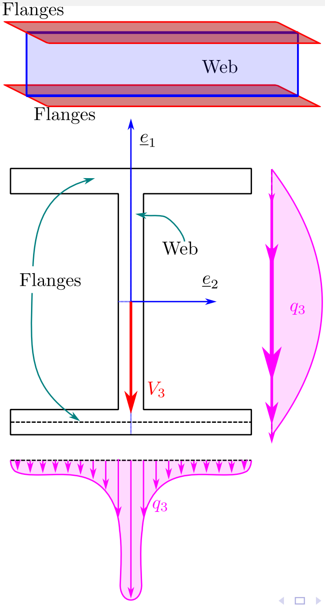
$$= \frac{tV_3}{2I_{22}} (X_2^2 - \frac{h^2}{4})$$

2. Shear Stress in Sections

- Consider the shear distribution through an I-section as shown here
- The shear distribution looks like it is "flowing", with more "flow" occurring in the thin vertical web and less in the flanges.
- If we consider the second moment of area I_{22}, it sums up as,

$$I_{22} = I_{web} + 2 \times I_{flange} = \frac{th^3}{12} + 2 \times \left(ht \times \frac{h^2}{4} \right)$$
$$= \underbrace{\frac{h^3 t}{12}}_{\approx 0} + \frac{h^3 t}{2},$$

which is dominated by the flanges.



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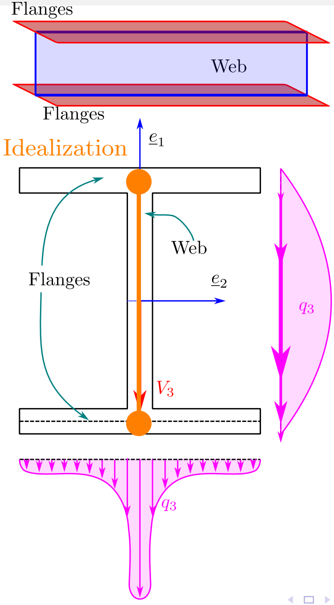
- It is therefore a useful approximation to lump the flange as a "point area" and the web as a "zero-area line".



2. Shear Stress in Sections

- Invoking plane stress assumption at the section, the governing equation is,

$$\sigma_{11,1} + \sigma_{1s,s} = 0.$$

- Shear flow formula

$$q(s) - q_0 = \frac{\left[\int_0^s t_D X_3 ds \quad -\int_0^s t_D X_2 ds\right]}{I_{22} I_{33} - I_{23}^2} \begin{bmatrix} I_{33} V_3 - I_{23} V_2 \\ I_{23} V_3 - I_{22} V_2 \end{bmatrix}$$

2. Shear Stress in Sections

- Point of shear load such that $\sum m_1 = 0$.

2. Shear Stress in Sections

- Choose constant of integration q_0 to exclude torsional effects.

2. Shear Stress in Sections

- If loads at boundary are self equilibrated to begin with, what is the distribution of loads in the member?

3. Stringer-Web Idealization

- Jump in shear flow across boom with area A_r:

$$q_2 - q_1 = -\sigma_{11,1} A_r$$

References I

[1] C. T. Sun. Mechanics of Aircraft Structures, 2nd edition. Hoboken, N.J: Wiley, June 2006. ISBN: 978-0-471-69966-8 (cit. on p. 2).

[2] T. H. G. Megson. Aircraft Structures for Engineering Students, Elsevier, 2013. ISBN: 978-0-08-096905-3 (cit. on p. 2).