AS3020*: Assignment 3

Module 3: Elasticity

Posted on 22-Aug-2024; Due at 11.59PM on 11-Sep-2024

General Instructions

1. Write this honor code and sign your name against it in the first page of your submission. Evaluation will not be done unless this is present in the submission.

   Upon my honor I state that I have received no unauthorized support and can attest that the submission reflects my understanding of the subject matter.

2. Discussions among students is permitted for this assignment. But ensure that your submission is your own. Do not write anything that you do not understand.

1 Answer in not more than two sentences 6

1. **(1)** What is the physical significance of the statement "a vector representing a material object is invariant under coordinate transformation"?

2. **(2)** What is the basis of the constraints necessary if you were to write down a constitutive relationship between stress components $\underset{\approx}{\sigma}$ and the deformation gradient $\underset{\approx}{F}$? (mention the most basic one discussed in class)

3. **(1)** As a ductile specimen is subjected to a Uniaxial Tensile Test, we have discussed that it undergoes failure through the formation/fusing of dislocations in the bulk (see fig. 1).

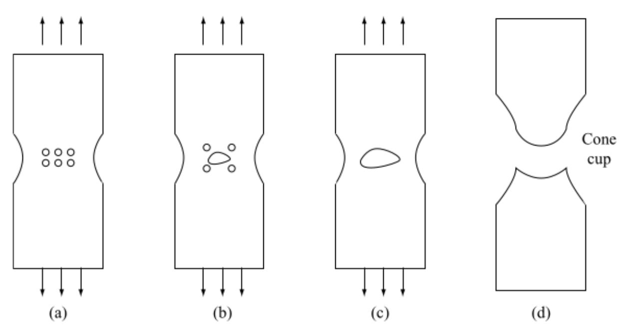


Figure 1: Ductile fracture from [1]

   Why does fracture not occur in a plane parallel to the ground, but instead, seems to tend towards a 45° plane? Assume isotropic elastic-perfectly plastic behavior.

   (If you deem it necessary, denote Young's modulus and Poisson's ratio as E, ν.) You can use up to 3 sentences for this.

4. **(2)** Give an example of a vector who's **components** are invariant under coordinate transformation, but the vector itself is not?

   Hint: A vector is always a representation of a measurement or an *experience*.

2 Answer Briefly 6

1. **(3)** I am interested in describing the deformation field $\underline{u}$ over a thin 2D flat plate, but I can only make strain measurements. Through instrumentation I obtain 3 independent measurements of the planar strain components $(E_{11}(\underline{x}), E_{12}(\underline{x}), E_{22}(\underline{x}))$.

   Since these are independent measurements, how can I check the quality of the data? Document your assumptions.

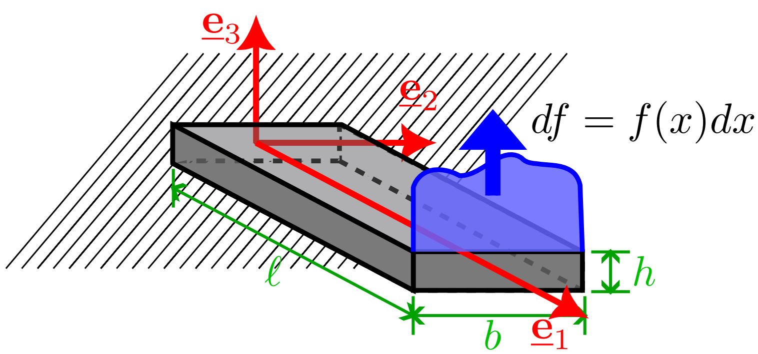
   Suppose I am interested in obtaining the displacement field as a function $\underline{u}(\underline{x})$ from this data, what are the equations I must solve? Explain in a single sentence how this may be solved.

2. **(3)** Consider the governing equations (in stress-based formulation) of 2D elasticity without body forces. Write down the governing equations in terms of the Airy stress functions.

   How does this relate to St. Venant's rule? Provide justification based on something you have seen in fluid mechanics/heat transfer.

3 Answer in Detail 8

1. **(4)** Consider the beam undergoing tip-line excitation as shown in item 1 below. Assume the origin is at center of the basis triad shown in the figure.



   Assume Euler-Bernoulli theory holds for this beam. Make any other simplifying assumptions and state them clearly. Use the following symbols where appropriate:

   - Section area A
   - Young's modulus E_y
   - Second moment of area I_y
   - Total force $F = \int_{-\frac{b}{2}}^{\frac{b}{2}} f(x)dx$

   Consider three points with reference position vectors:

   - $P_1 : \frac{\ell}{4}\underline{e}_1$
   - $P_2 : \frac{\ell}{4}\underline{e}_1 + \frac{h}{2}\underline{e}_3$
   - $P_3 : \frac{\ell}{4}\underline{e}_1 + \frac{b}{4}\underline{e}_2 + \frac{h}{4}\underline{e}_3$

   Write out the following for each of the above (at equilibrium):

   - Deformed position vector
   - 3D Stress tensor
   - Principal axes of stress
   - 3D (infinitesimal) Strain tensor
   - Principal axes of strain

2. **(4)** Derive an expression for how an infinitesimal area in the reference gets transformed by filling up the following.

   - We will choose two vectors $d\underline{X}^{(1)}$, $d\underline{X}^{(2)}$ in the undeformed coordinates. The area vector defined by these two is written as,

   $$d\underline{A} = d\underline{X}^{(1)} \times d\underline{X}^{(2)} = (\ldots\ldots\ldots)\underline{e}_I$$

   - In the deformed configuration, these deform as (use $F_{iI} = \frac{\partial x_i}{\partial x_I}$),

   $$d\underline{x}^{(1)} = (\ldots\ldots\ldots)\underline{e}_i \tag{1a}$$
   $$d\underline{x}^{(2)} = (\ldots\ldots\ldots)\underline{e}_j \tag{1b}$$

   - The deformed area is written as,

   $$d\underline{a} = (\ldots\ldots\ldots) = (\ldots\ldots\ldots)\underline{e}_i.$$

   Substituting eq. (1) this becomes,

   $$d\underline{a} = da_i\underline{e}_i = (\ldots\ldots\ldots)dX_J^{(1)}dX_K^{(2)}\underline{e}_i \tag{2}$$

   - We make a substitution of $\underline{e}_i = \delta_{im}\underline{e}_m$ in eq. (2) to get,

   $$d\underline{a} = (\ldots\ldots\ldots)dX_J^{(1)}dX_K^{(2)}\underline{e}_m.$$

   We now write $\delta_{im} = F_{iI}(\mathbb{F}^{-1})_{Im}$, which leads to,

   $$\begin{aligned} d\underline{a} &= \epsilon_{ijk}(\ldots\ldots\ldots)\underline{e}_m \\ &= det(\mathbb{F})(\mathbb{F}^{-1})_{Im}(\ldots\ldots\ldots)\underline{e}_m \\ &= (\ldots\ldots\ldots)\underline{e}_m \qquad \text{(final simplification).} \end{aligned} \tag{3}$$

   - Equation (3) is expressed in <u>vector component notation</u> (using arrays) as

   $$d\underset{\sim}{a} = (\ldots\ldots\ldots)d\underset{\sim}{A}.$$

References

[1] V Rajendran. *Materials Science*. Tata McGraw-Hill Education.