Point E

For the Gerber criterion, from Eq. (8–46), the safety factor is

Answer

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e(S_e + \sigma_i)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2(3.10)(18.6)}\left[120\sqrt{120^2 + 4(18.6)(18.6 + 63.72)} - 120^2 - 2(63.72)(18.6)\right]$$

$$= 3.65$$

which is greater than $n_p = 3.43$ and contradicts the conclusion earlier that the danger of failure is fatigue. Figure 8–22 clearly shows the conflict where point D lies between points C and E. Again, the conservative nature of the Goodman criterion explains the discrepancy and the designer must form his or her own conclusion.

8–12 Bolted and Riveted Joints Loaded in Shear[10]

Riveted and bolted joints loaded in shear are treated exactly alike in design and analysis.

Figure 8–23*a* shows a riveted connection loaded in shear. Let us now study the various means by which this connection might fail.

Figure 8–23*b* shows a failure by bending of the rivet or of the riveted members. The bending moment is approximately $M = Ft/2$, where F is the shearing force and t is the grip of the rivet, that is, the total thickness of the connected parts. The bending stress in the members or in the rivet is, neglecting stress concentration,

$$\sigma = \frac{M}{I/c} \qquad (8\text{–}52)$$

where I/c is the section modulus for the weakest member or for the rivet or rivets, depending upon which stress is to be found. The calculation of the bending stress in this manner is an assumption, because we do not know exactly how the load is distributed to the rivet or the relative deformations of the rivet and the members. Although this equation can be used to determine the bending stress, it is seldom used in design; instead its effect is compensated for by an increase in the factor of safety.

In Fig. 8–23*c* failure of the rivet by pure shear is shown; the stress in the rivet is

$$\tau = \frac{F}{A} \qquad (8\text{–}53)$$

where A is the cross-sectional area of all the rivets in the group. It may be noted that it is standard practice in structural design to use the nominal diameter of the rivet rather than the diameter of the hole, even though a hot-driven rivet expands and nearly fills up the hole.

[10]The design of bolted and riveted connections for boilers, bridges, buildings, and other structures in which danger to human life is involved is strictly governed by various construction codes. When designing these structures, the engineer should refer to the *American Institute of Steel Construction Handbook,* the American Railway Engineering Association specifications, or the Boiler Construction Code of the American Society of Mechanical Engineers.

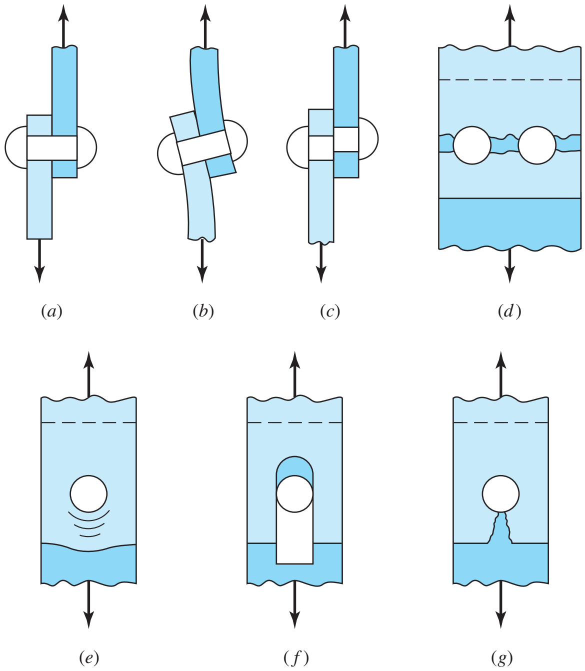


Figure 8–23

Modes of failure in shear loading of a bolted or riveted connection: (*a*) shear loading; (*b*) bending of rivet; (*c*) shear of rivet; (*d*) tensile failure of members; (*e*) bearing of rivet on members or bearing of members on rivet; (*f*) shear tear-out; (*g*) tensile tear-out.

Rupture of one of the connected members or plates by pure tension is illustrated in Fig. 8–23*d*. The tensile stress is

$$\sigma = \frac{F}{A} \tag{8–54}$$

where A is the net area of the plate, that is, the area reduced by an amount equal to the area of all the rivet holes. For brittle materials and static loads and for either ductile or brittle materials loaded in fatigue, the stress-concentration effects must be included. It is true that the use of a bolt with an initial preload and, sometimes, a rivet will place the area around the hole in compression and thus tend to nullify the effects of stress concentration, but unless definite steps are taken to ensure that the preload does not relax, it is on the conservative side to design as if the full stress-concentration effect were present. The stress-concentration effects are not considered in structural design, because the loads are static and the materials ductile.

In calculating the area for Eq. (8–54), the designer should, of course, use the combination of rivet or bolt holes that gives the smallest area.

Figure 8–23*e* illustrates a failure by crushing of the rivet or plate. Calculation of this stress, which is usually called a *bearing stress,* is complicated by the distribution of the load on the cylindrical surface of the rivet. The exact values of the forces acting upon the rivet are unknown, and so it is customary to assume that the components of these forces are uniformly distributed over the projected contact area of the rivet. This gives for the stress

$$\sigma = -\frac{F}{A} \tag{8–55}$$

where the projected area for a single rivet is $A = td$. Here, t is the thickness of the thinnest plate and d is the rivet or bolt diameter.

Edge shearing, or tearing, of the margin is shown in Fig. 8–23*f* and *g*, respectively. In structural practice this failure is avoided by spacing the rivets at least $1\frac{1}{2}$ diameters away from the edge. Bolted connections usually are spaced an even greater distance than this for satisfactory appearance, and hence this type of failure may usually be neglected.

In a rivet joint, the rivets all share the load in shear, bearing in the rivet, bearing in the member, and shear in the rivet. Other failures are participated in by only some of the joint. In a bolted joint, shear is taken by clamping friction, and bearing does not exist. When bolt preload is lost, one bolt begins to carry the shear and bearing until yielding slowly brings other fasteners in to share the shear and bearing. Finally, all participate, and this is the basis of most bolted-joint analysis if loss of bolt preload is complete. The usual analysis involves

- Bearing in the bolt (all bolts participate)
- Bearing in members (all holes participate)
- Shear of bolt (all bolts participate eventually)
- Distinguishing between thread and shank shear
- Edge shearing and tearing of member (edge bolts participate)
- Tensile yielding of member across bolt holes
- Checking member capacity

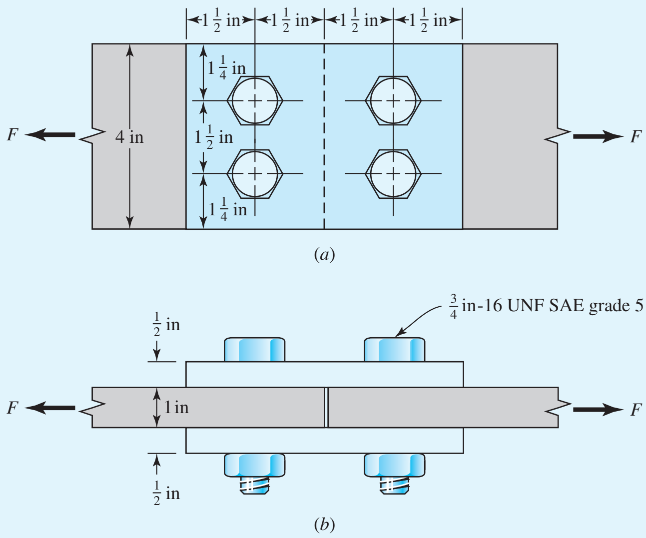
EXAMPLE 8–6

Two 1- by 4-in 1018 cold-rolled steel bars are butt-spliced with two $\frac{1}{2}$- by 4-in 1018 cold-rolled splice plates using four $\frac{3}{4}$ in-16 UNF grade 5 bolts as depicted in Fig. 8–24. For a design factor of $n_d = 1.5$ estimate the static load F that can be carried if the bolts lose preload.

Solution

From Table A–20, minimum strengths of $S_y = 54$ kpsi and $S_{ut} = 64$ kpsi are found for the members, and from Table 8–9 minimum strengths of $S_p = 85$ kpsi, $S_y = 92$ kpsi, and $S_{ut} = 120$ kpsi for the bolts are found.

Figure 8–24



$F/2$ is transmitted by each of the splice plates, but since the areas of the splice plates are half those of the center bars, the stresses associated with the plates are the same. So for stresses associated with the plates, the force and areas used will be those of the center plates.

Bearing in bolts, all bolts loaded:

$$\sigma = \frac{F}{2td} = \frac{S_y}{n_d}$$

$$F = \frac{2td\,S_y}{n_d} = \frac{2(1)(\frac{3}{4})92}{1.5} = 92 \text{ kip}$$

Bearing in members, all bolts active:

$$\sigma = \frac{F}{2td} = \frac{(S_y)_{\text{mem}}}{n_d}$$

$$F = \frac{2td(S_y)_{\text{mem}}}{n_d} = \frac{2(1)(\frac{3}{4})54}{1.5} = 54 \text{ kip}$$

Shear of bolt, all bolts active: If the bolt threads do not extend into the shear planes for four shanks:

$$\tau = \frac{F}{4\pi d^2/4} = 0.577\frac{S_y}{n_d}$$

$$F = 0.577\pi d^2\frac{S_y}{n_d} = 0.577\pi(0.75)^2\frac{92}{1.5} = 62.5 \text{ kip}$$

If the bolt threads extend into a shear plane:

$$\tau = \frac{F}{4A_r} = 0.577\frac{S_y}{n_d}$$

$$F = \frac{0.577(4)A_r S_y}{n_d} = \frac{0.577(4)0.351(92)}{1.5} = 49.7 \text{ kip}$$

Edge shearing of member at two margin bolts: From Fig. 8–25,

$$\tau = \frac{F}{4at} = \frac{0.577(S_y)_{\text{mem}}}{n_d}$$

$$F = \frac{4at0.577(S_y)_{\text{mem}}}{n_d} = \frac{4(1.125)(1)0.577(54)}{1.5} = 93.5 \text{ kip}$$

Tensile yielding of members across bolt holes:

$$\sigma = \frac{F}{[4 - 2(\frac{3}{4})]t} = \frac{(S_y)_{\text{mem}}}{n_d}$$

$$F = \frac{[4 - 2(\frac{3}{4})]t(S_y)_{\text{mem}}}{n_d} = \frac{[4 - 2(\frac{3}{4})](1)54}{1.5} = 90 \text{ kip}$$

On the basis of bolt shear, the limiting value of the force is 49.7 kip, assuming the threads extend into a shear plane. However, it would be poor design to allow the threads to extend into a shear plane. So, assuming a *good* design based on bolt shear, the limiting value of the force is 62.5 kip. For the members, the bearing stress limits the load to 54 kip.

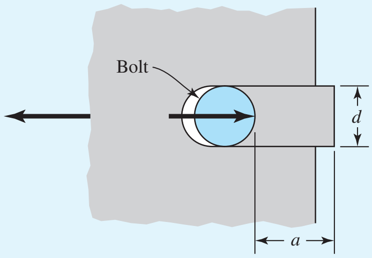


Figure 8–25

Edge shearing of member.

Shear Joints with Eccentric Loading

In the previous example, the load distributed equally to the bolts since the load acted along a line of symmetry of the fasteners. The analysis of a shear joint undergoing eccentric loading requires locating the center of relative motion between the two members. In Fig. 8–26 let A_1 to A_5 be the respective cross-sectional areas of a group of five pins, or hot-driven rivets, or tight-fitting shoulder bolts. Under this assumption the rotational pivot point lies at the centroid of the cross-sectional area pattern of the pins, rivets, or bolts. Using statics, we learn that the centroid G is located by the coordinates $\bar{x}$ and $\bar{y}$, where x_i and y_i are the distances to the ith area center:

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4 + A_5x_5}{A_1 + A_2 + A_3 + A_4 + A_5} = \frac{\sum_1^n A_ix_i}{\sum_1^n A_i}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3 + A_4y_4 + A_5y_5}{A_1 + A_2 + A_3 + A_4 + A_5} = \frac{\sum_1^n A_iy_i}{\sum_1^n A_i} \qquad (8\text{–}56)$$

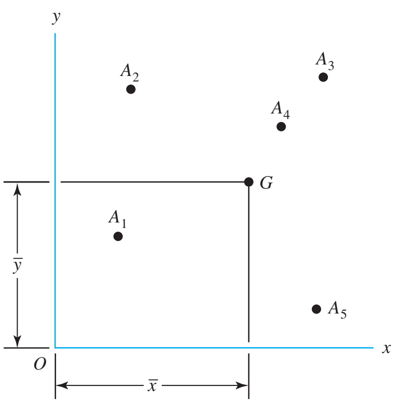


Figure 8–26

Centroid of pins, rivets, or bolts.

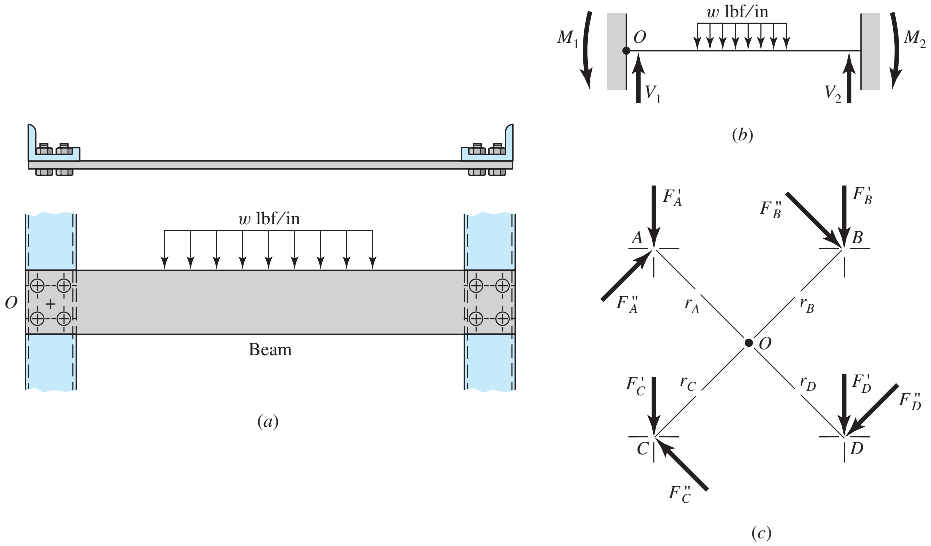


Figure 8–27

(*a*) Beam bolted at both ends with distributed load; (*b*) free-body diagram of beam; (*c*) enlarged view of bolt group centered at *O* showing primary and secondary resultant shear forces.

In many instances the centroid can be located by symmetry.

An example of eccentric loading of fasteners is shown in Fig. 8–27. This is a portion of a machine frame containing a beam subjected to the action of a bending load. In this case, the beam is fastened to vertical members at the ends with specially prepared load-sharing bolts. You will recognize the schematic representation in Fig. 8–27*b* as a statically indeterminate beam with both ends fixed and with moment and shear reactions at each end.

For convenience, the centers of the bolts at the left end of the beam are drawn to a larger scale in Fig. 8–27*c*. Point *O* represents the centroid of the group, and it is assumed in this example that all the bolts are of the same diameter. Note that the forces shown in Fig. 8–27*c* are the *resultant* forces acting on the pins with a net force and moment equal and opposite to the *reaction* loads V_1 and M_1 acting at *O*. The total load taken by each bolt will be calculated in three steps. In the first step the shear V_1 is divided equally among the bolts so that each bolt takes $F' = V_1/n$, where n refers to the number of bolts in the group and the force F' is called the *direct load,* or *primary shear*.

It is noted that an equal distribution of the direct load to the bolts assumes an absolutely rigid member. The arrangement of the bolts or the shape and size of the members sometimes justifies the use of another assumption as to the division of the load. The direct loads F'_n are shown as vectors on the loading diagram (Fig. 8–27*c*).

The *moment load,* or *secondary shear,* is the additional load on each bolt due to the moment M_1. If r_A, r_B, r_C, etc., are the radial distances from the centroid to the center of each bolt, the moment and moment loads are related as follows:

$$M_1 = F''_A r_A + F''_B r_B + F''_C r_C + \cdots \tag{a}$$

where the F'' are the moment loads. The force taken by each bolt depends upon its radial distance from the centroid; that is, the bolt farthest from the centroid takes the greatest load, while the nearest bolt takes the smallest. We can therefore write

$$\frac{F''_A}{r_A} = \frac{F''_B}{r_B} = \frac{F''_C}{r_C} \tag{b}$$

where again, the diameters of the bolts are assumed equal. If not, then one replaces F'' in Eq. (b) with the shear stresses $\tau'' = 4F''/\pi d^2$ for each bolt. Solving Eqs. (a) and (b) simultaneously, we obtain

$$F_n'' = \frac{M_1 r_n}{r_A^2 + r_B^2 + r_C^2 + \cdots} \tag{8–57}$$

where the subscript n refers to the particular bolt whose load is to be found. Each moment load is a force vector perpendicular to the radial line from the centroid to the bolt center.

In the third step the direct and moment loads are added vectorially to obtain the resultant load on each bolt. Since all the bolts or rivets are usually the same size, only that bolt having the maximum load need be considered. When the maximum load is found, the strength may be determined by using the various methods already described.

EXAMPLE 8–7

Shown in Fig. 8–28 is a 15- by 200-mm rectangular steel bar cantilevered to a 250-mm steel channel using four tightly fitted bolts located at A, B, C, and D. Assume the bolt threads do not extend into the joint.

For the $F = 16$ kN load shown find
(a) The resultant load on each bolt
(b) The maximum shear stress in each bolt
(c) The maximum bearing stress
(d) The critical bending stress in the bar

Solution

(a) Point O, the centroid of the bolt group in Fig. 8–28, is found by symmetry. If a free-body diagram of the beam were constructed, the shear reaction V would pass through O and the moment reactions M would be about O. These reactions are

$$V = 16 \text{ kN} \qquad M = 16(300 + 50 + 75) = 6800 \text{ N} \cdot \text{m}$$

In Fig. 8–29, the bolt group has been drawn to a larger scale and the reactions and resultants are shown. The distance from the centroid to the center of each bolt is

$$r = \sqrt{(60)^2 + (75)^2} = 96.0 \text{ mm}$$

Figure 8–28

Dimensions in millimeters.

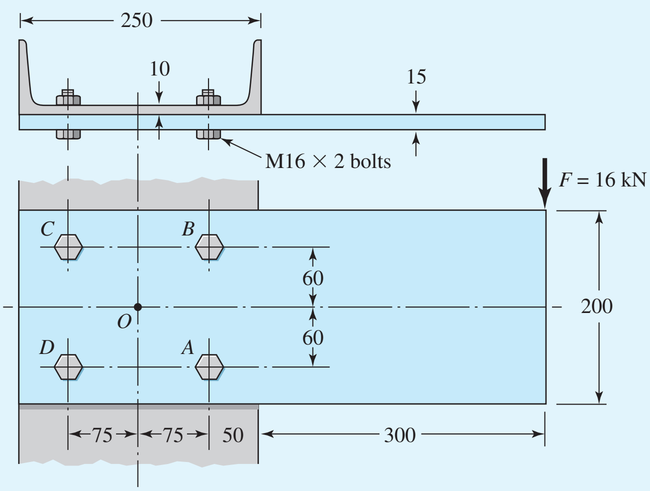
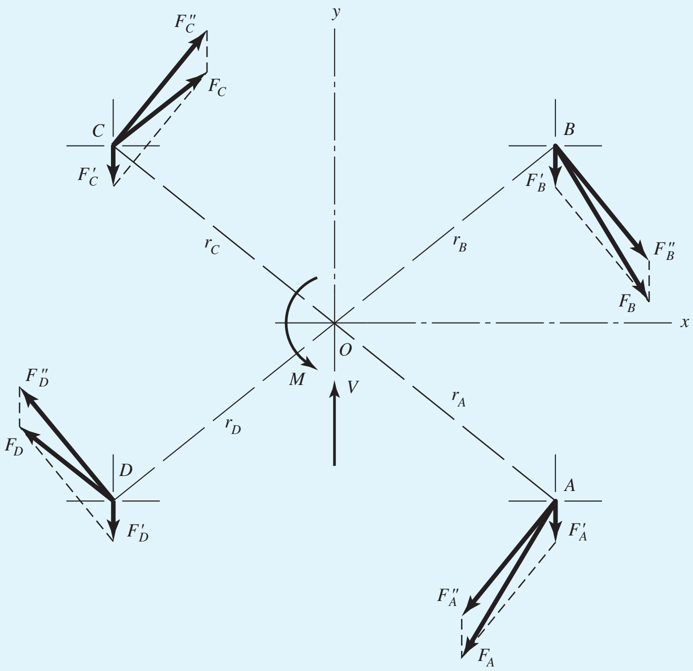


Figure 8–29



The resultants are found as follows. The primary shear load per bolt is

$$F' = \frac{V}{n} = \frac{16}{4} = 4 \text{ kN}$$

Since the r_n are equal, the secondary shear forces are equal, and Eq. (8–57) becomes

$$F'' = \frac{Mr}{4r^2} = \frac{M}{4r} = \frac{6800}{4(96.0)} = 17.7 \text{ kN}$$

The primary and secondary shear forces are plotted to scale in Fig. 8–29 and the resultants obtained by using the parallelogram rule. The magnitudes are found by measurement (or analysis) to be

Answer $$F_A = F_B = 21.0 \text{ kN}$$

Answer $$F_C = F_D = 14.8 \text{ kN}$$

(*b*) Bolts *A* and *B* are critical because they carry the largest shear load. The problem stated to assume that the bolt threads are not to extend into the joint. This would require special bolts. If standard nuts and bolts were used, the bolts would need to be 46 mm long with a thread length of $L_T = 38$ mm. Thus the unthreaded portion of the bolt is $46 - 38 = 8$ mm long. This is less than the 15 mm for the plate in Fig. 8–28, and the bolts would tend to shear along the minor diameter at a stress of $\tau = F/A_s = 21.0(10)^3/144 = 146$ MPa. Using bolts not extending into the joint, or shoulder bolts, is preferred. For this example, the body area of each bolt is $A = \pi(16^2)/4 = 201.1 \text{ mm}^2$, resulting in a shear stress of

Answer $$\tau = \frac{F}{A} = \frac{21.0(10)^3}{201.1} = 104 \text{ MPa}$$

(c) The channel is thinner than the bar, and so the largest bearing stress is due to the pressing of the bolt against the channel web. The bearing area is $A_b = td = 10(16) = 160\ \text{mm}^2$. Thus the bearing stress is

Answer
$$\sigma = -\frac{F}{A_b} = -\frac{21.0(10)^3}{160} = -131\ \text{MPa}$$

(d) The critical bending stress in the bar is assumed to occur in a section parallel to the y axis and through bolts A and B. At this section the bending moment is

$$M = 16(300 + 50) = 5600\ \text{N}\cdot\text{m}$$

The second moment of area through this section is obtained as follows:

$$I = I_{\text{bar}} - 2(I_{\text{holes}} + \bar{d}^2 A)$$
$$= \frac{15(200)^3}{12} - 2\left[\frac{15(16)^3}{12} + (60)^2(15)(16)\right] = 8.26(10)^6\ \text{mm}^4$$

Then

Answer
$$\sigma = \frac{Mc}{I} = \frac{5600(100)}{8.26(10)^6}(10)^3 = 67.8\ \text{MPa}$$

PROBLEMS

8–1 A power screw is 25 mm in diameter and has a thread pitch of 5 mm.
(a) Find the thread depth, the thread width, the mean and root diameters, and the lead, provided square threads are used.
(b) Repeat part (a) for Acme threads.

8–2 Using the information in the footnote of Table 8–1, show that the tensile-stress area is

$$A_t = \frac{\pi}{4}(d - 0.938\,194p)^2$$

8–3 Show that for zero collar friction the efficiency of a square-thread screw is given by the equation

$$e = \tan\lambda \frac{1 - f\tan\lambda}{\tan\lambda + f}$$

Plot a curve of the efficiency for lead angles up to 45°. Use $f = 0.08$.

8–4 A single-threaded power screw is 25 mm in diameter with a pitch of 5 mm. A vertical load on the screw reaches a maximum of 5 kN. The coefficients of friction are 0.06 for the collar and 0.09 for the threads. The frictional diameter of the collar is 45 mm. Find the overall efficiency and the torque to "raise" and "lower" the load.

8–5 The machine shown in the figure can be used for a tension test but not for a compression test. Why? Can both screws have the same hand?