AS2070: Aerospace Structural Mechanics

Module 2: Composite Material Mechanics

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Dept. of Aerospace Engg., IIT Madras, Chennai

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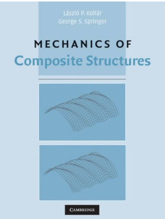
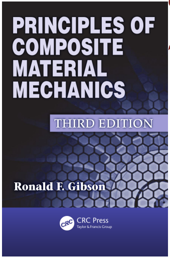
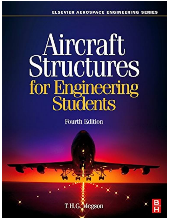
(Also see Daniel and Ishai 2006)



Chapters 1-3, 11 in Kollár and Springer (2003).



Chapters 1-3 in Gibson (2012).

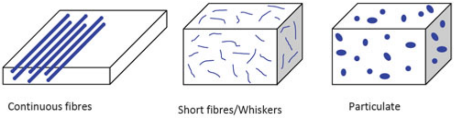


Chapter 25 in Megson (2013)

1.1. What are Composites?

Introduction

- Structural material consisting of multiple non-soluble macro-constituents.
- Main motivation: material properties tailored to applications.
- Both stiffness and strength comes from the fibers/particles, and the matrix holdes everything together.



Types of composite materials (*Figure from NPTEL Online-IIT KANPUR (2025)*)

Examples

- Reinforced concrete
- Wood (lignin matrix reinforced by cellulose fibers)
- Carbon-Fiber Reinforced Plastics (CFRP)

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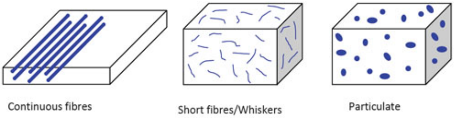

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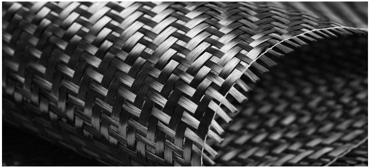
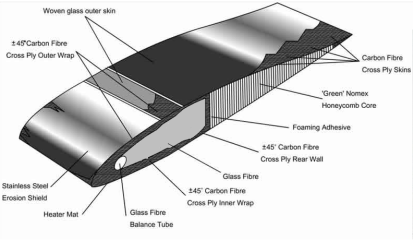


1.1. What are Composites?

Introduction

- Structural material consisting of multiple non-soluble macro-constituents.

CFRP Helicopter Blades



(Figures from Carbon Fiber Top Helicopter Blades 2025)

Ex

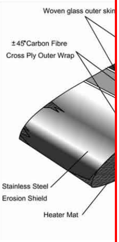
- Reinforced concrete
- Wood (lignin matrix reinforced by cellulose fibers)
- Carbon-Fiber Reinforced Plastics (CFRP)

- ~2x stiffness, ~3x strength, ~ 70% weight of AA.
- High fatigue resistance. But quite brittle.
- Main- and tail-planes, fuselages, etc. Helicopter blades.

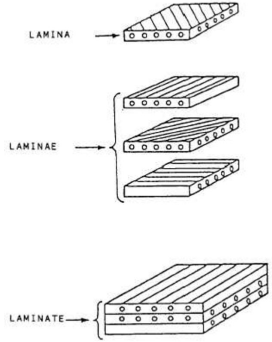
1.1. What are Composites?

Introduction

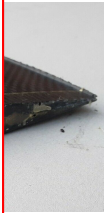
- Structural material consisting of multiple non-soluble macro-constituents.



Laminated Composites



(Figure from Kalkan 2017)



Ex

- Reinforced co
- Wood (lignin
  cellulose fiber
- Carbon-Fiber Reinforced Plastics (CFRP)

gth, ∼ 70%

But quite brittle.

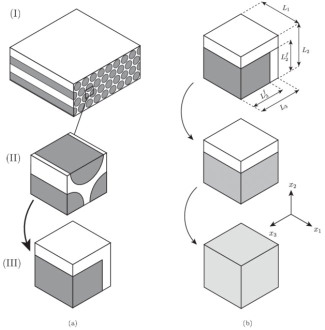
- Main- and tail-planes, fuselages, etc. Helicopter blades.

1.2. Modeling Composite Material

Introduction

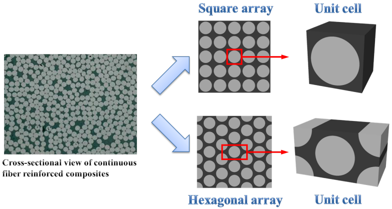
Two main approaches:

Macro-Mechanics



Homogenization of micro-structure *(Figure from Skovsgaard and Heide-Jørgensen 2021)*

Micro-Mechanics



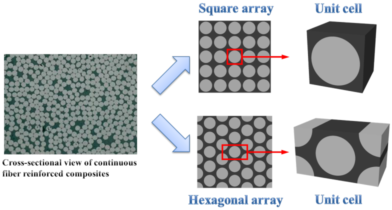
(Figure from "Micro-Mechanics of Failure" 2024)

1.2. Modeling Composite Material

Introduction

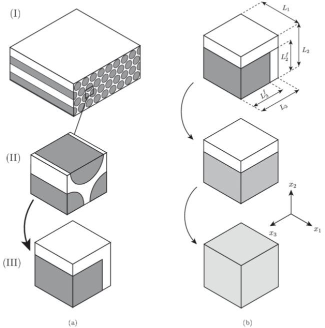
Two main approaches:

Micro-Mechanics



(Figure from "Micro-Mechanics of Failure" 2024)

Macro-Mechanics ✓

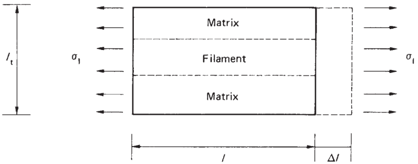


Homogenization of micro-structure (Figure from Skovsgaard and Heide-Jørgensen 2021)

1.3. Constitutive Modeling for Composites

Introduction

Axial Elongation



- Strain is fixed, but stress experienced by media differ.

$$\sigma_l = E_l \varepsilon_l$$

- Stress-strain relationship simplifies as,

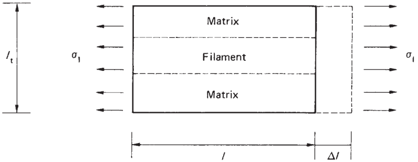
$$\sigma_m = E_m \varepsilon_l, \quad \sigma_f = E_f \varepsilon_l$$
$$\sigma_l A = \sigma_m A_m + \sigma_f A_f$$
$$\implies \boxed{E_l = \frac{A_f}{A} E_f + \frac{A_m}{A} E_m}.$$

(Figures from Megson 2013)

1.3. Constitutive Modeling for Composites

Introduction

Axial Elongation



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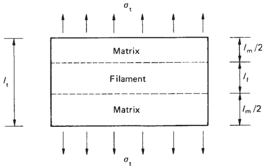
Transverse Elongation



- Stress is fixed, strains differ:

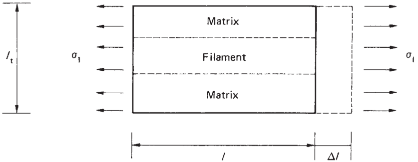
$$\varepsilon_t l_t = \varepsilon_m l_m + \varepsilon_f l_f$$
$$\implies \frac{\sigma_t}{E_t} l_t = \frac{\sigma_t}{E_m} l_m + \frac{\sigma_t}{E_f} l_f$$
$$\implies \boxed{\frac{1}{E_t} = \frac{1}{E_m}\frac{l_m}{l_t} + \frac{1}{E_f}\frac{l_f}{l_t}}.$$

(Figures from Megson 2013)

1.3. Constitutive Modeling for Composites

Introduction: Poisson Effects

Axial-Transverse Coupling



- Transverse displacement written as

$$\Delta_t = \nu_m \varepsilon_l l_m + \nu_f \varepsilon_l l_f := \nu_{lt} \varepsilon_l l_t$$

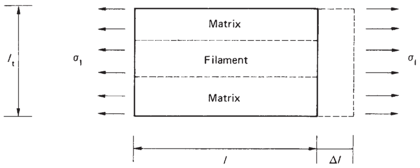
$$\implies \boxed{\nu_{lt} = \frac{l_m}{l_t}\varepsilon_l + \frac{l_f}{l_t}\varepsilon_f}.$$

(Figures from Megson 2013)

1.3. Constitutive Modeling for Composites

Introduction: Poisson Effects

Axial-Transverse Coupling



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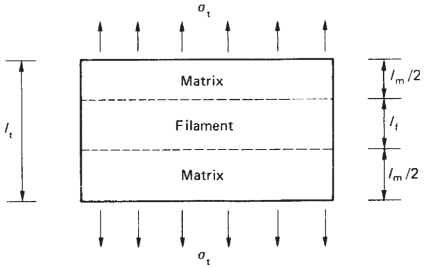
Transverse-Axial Coupling



- Axial displacement written as

$$\nu_m \frac{\sigma_t}{E_m} = \nu_f \frac{\sigma_t}{E_f} := \nu_{tl} \frac{\sigma_t}{E_t},$$

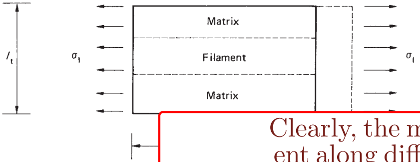
$$\implies \boxed{\nu_{tl} = \frac{E_t}{E_l}\nu_{lt}}.$$

(Figures from Megson 2013)

1.3. Constitutive Modeling for Composites

Introduction: Poisson Effects

Axial-Transverse Coupling



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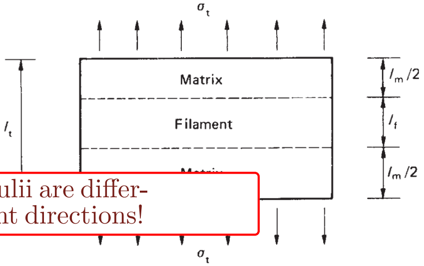
Transverse-Axial Coupling



- Axial displacement written as

$$\nu_m \frac{\sigma_t}{E_m} = \nu_f \frac{\sigma_t}{E_f} := \nu_{tl} \frac{\sigma_t}{E_t},$$

$$\implies \boxed{\nu_{tl} = \frac{E_t}{E_l}\nu_{lt}}.$$

Clearly, the modulii are different along different directions!

(Figures from Megson 2013)

1.3. Constitutive Modeling for Composites

Introduction: Anisotropy

General Anisotropy

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16}\\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26}\\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36}\\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46}\\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56}\\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66}\end{bmatrix} \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix}$$

1.3. Constitutive Modeling for Composites

Introduction: Anisotropy

General Anisotropy

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Monoclinic: Single Plane of Symmetry

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & 0 & 0\\ C_{12} & C_{22} & C_{23} & C_{24} & 0 & 0\\ C_{13} & C_{23} & C_{33} & C_{34} & 0 & 0\\ C_{14} & C_{24} & C_{34} & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & C_{56}\\ 0 & 0 & 0 & 0 & C_{56} & C_{66}\end{bmatrix} \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix}$$

1.3. Constitutive Modeling for Composites

Introduction: Anisotropy

Triclinic: Three Planes of Symmetry

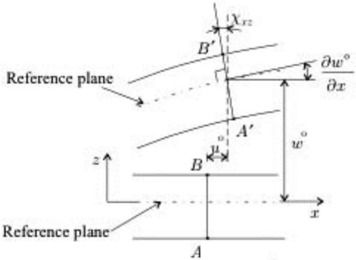
$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix} \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix}$$

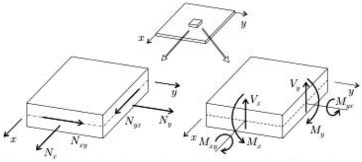
Transversely Isotropic

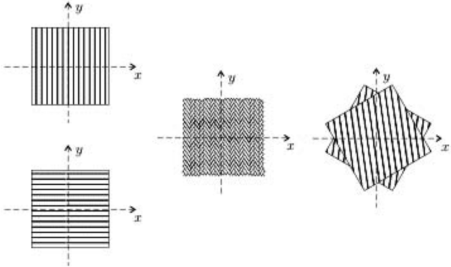
$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{13} & 0 & 0 & 0\\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2}\end{bmatrix} \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix}$$

1.4. Classical Laminate Theory

Introduction

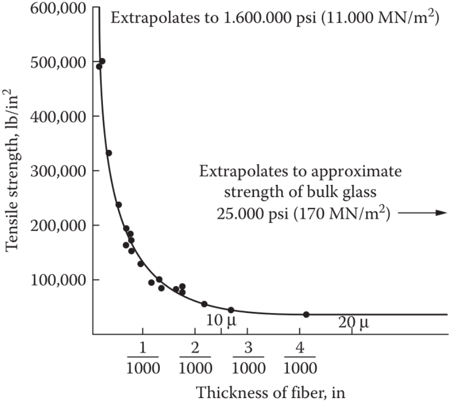






Figures from Kollár and Springer 2003

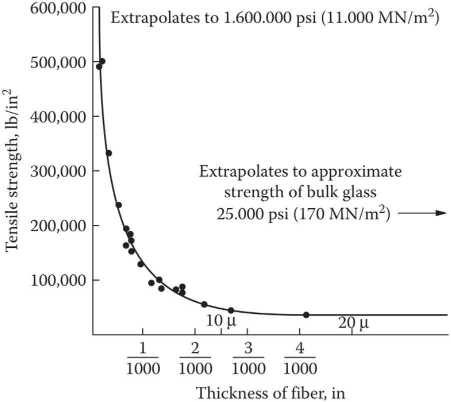
2. Composite Materials



Griffith's experiments with glass fibres (1920)

(Figure from Gibson 2012)

2. Composite Materials



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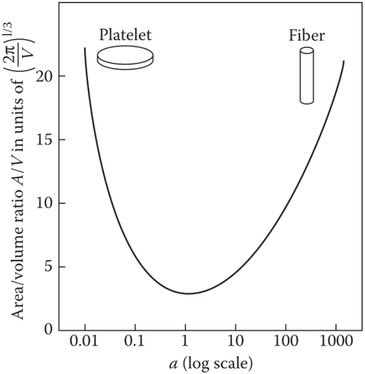


(Figure from Gibson 2012)

2.1. Types of Composite Materials

Composite Materials

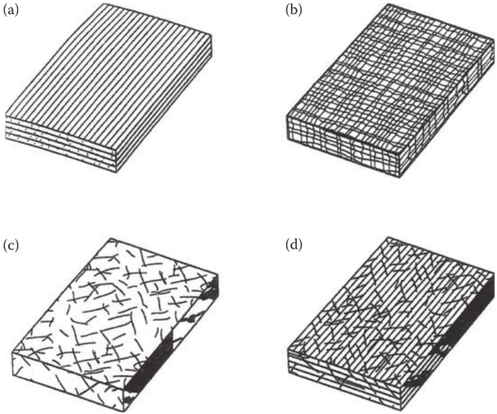


FIGURE 1.4
Types of fiber-reinforced composites. (a) Continuous fiber composite, (b) woven composite, (c) chopped fiber composite, and (d) hybrid composite.

(Figure from Gibson 2012)

3.1. The Rule of Mixtures

Micro-Mechanics Descriptions

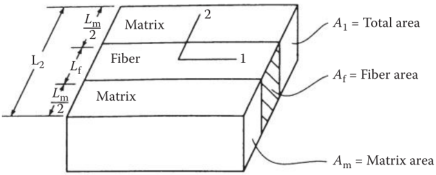
The rule of mixtures is introduced as a very simple framework for developing "overall"/representative mechanical properties.

Basic Definitions

Subscripts $(\cdot)_f$, $(\cdot)_m$, $(\cdot)_v$, and $(\cdot)_c$ denote quantities corresponding to the fiber, matrix, void, and composite (as a whole).

Volume Fraction $v_f = \frac{V_f}{V_c}, v_m = \frac{V_m}{V_c}, v_v = \frac{V_v}{V_c}$ such that $v_f + v_m + v_v = 1$.
Note that composite density $\rho_c = \rho_f v_f + \rho_m v_m$.

Weight Fraction $w_f = \frac{\rho_f}{\rho_c} v_f$



(Figure 3.5a from Gibson 2012)

$$E_1 = v_f E_f + v_m E_m$$

$$(\times) E_2 = \left(\frac{v_f}{E_f} + \frac{v_m}{E_m}\right)^{-1}$$

$$\nu_{12} = v_f \nu_f + v_m \nu_m$$

$$(\times) G_{12} = \left(\frac{v_f}{G_f} + \frac{v_m}{G_m}\right)^{-1}$$

3.1. The Rule of Mixtures

Micro-Mechanics Descriptions

The rule of mixtures is introduced as a very simple framework for developing "overal

Basic

Subscri ... e fiber, matrix,

Volum ... = 1.

Weigh

RoM is not always satisfactory!

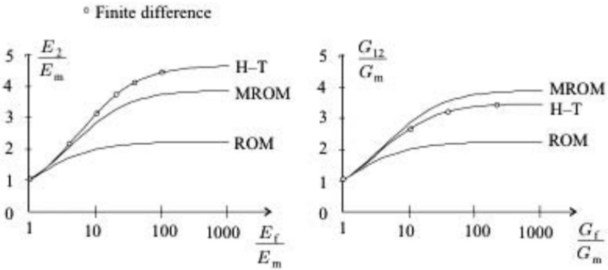
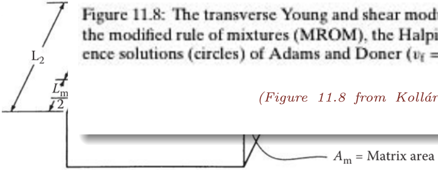


Figure 11.8: The transverse Young and shear moduli calculated by the rule of mixtures (ROM), the modified rule of mixtures (MROM), the Halpin–Tsai (H–T) equations, and the finite difference solutions (circles) of Adams and Doner ($v_f = 0.55$).

(Figure 11.8 from Kollár and Springer 2003)



(Figure 3.5a from Gibson 2012)

E_m

$)^{-1}$

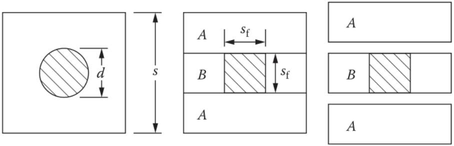
ν_m

$$(\times)G_{12} = \left(\frac{v_f}{G_f} + \frac{v_m}{G_m}\right)^{-1}$$

3.1. The Rule of Mixtures

Micro-Mechanics Descriptions

- The mismatch is related to the fact that our idealized picture was a poor representation of reality to begin with. More geometrical details of the fiber arrangement are necessary.



(Figure 3.8 from Gibson 2012)

$$s_f = \sqrt{\frac{\pi}{4}} d; s = \sqrt{\frac{\pi}{4v_f}} d.$$

$$\begin{aligned} E_{B2} &= \left(\frac{\sqrt{v_f}}{E_f} + \frac{1-\sqrt{v_f}}{E_m} \right)^{-1} \\ &= \frac{E_m}{1-\sqrt{v_f}(1-\frac{E_m}{E_f})} \end{aligned}$$

$$\begin{aligned} E_2 &= E_{B2}\frac{s_f}{s} + E_m \frac{s-s_f}{s} \\ &= E_{B2}\sqrt{v_f} + E_m(1-\sqrt{v_f}) \\ &= E_m \left[(1-\sqrt{v_f}) + \frac{\sqrt{v_f}}{1-\sqrt{v_f}(1-\frac{E_m}{E_f})} \right] \end{aligned}$$

3.1. The Rule of Mixtures

Micro-Mechanics Descriptions

(Recommended reading: Sec. 3.2.3 in Daniel and Ishai 2006)

The Halpin-Tsai Equation

$$E_2 = E_m \frac{1+\xi\eta v_f}{1-\eta v_f}, \quad \eta = \frac{E_f - E_m}{E_f + \xi E_m}$$
$$= E_m \frac{E_f + \xi E_m + \xi v_f(E_f - E_m)}{E_f + \xi E_m - v_f(E_f - E_m)}$$

Note: $\xi = 2$ for circular section fibers. $\xi = \frac{2a}{b}$ for rectangular fibers (b being loaded side).

Case 1: $\xi \to 0$

$$E_2 = \left(\frac{v_f}{E_f} + \frac{1-v_f}{E_m}\right)^{-1}$$

Series, *Reuss* model.

Case 2: $\xi \to \infty$

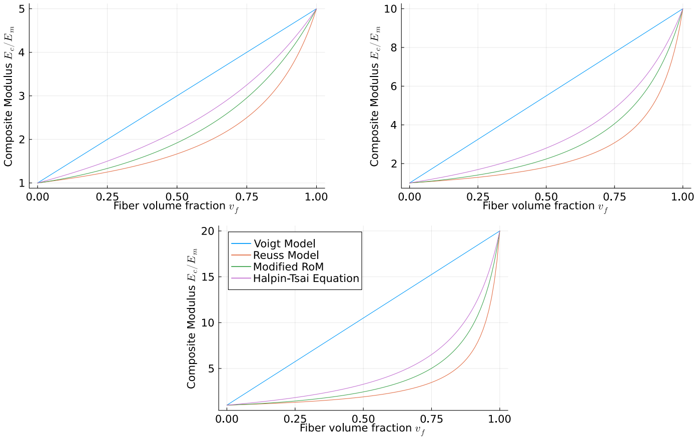
$$E_2 = E_f v_f + E_m(1-v_f)$$

Parallel, *Voigt* model.

3.1. The Rule of Mixtures

Micro-Mechanics Descriptions

Graphical Comparison for varying $\frac{E_f}{E_m}$



ishai 2006)

Note: ... loaded side).

Series, Reuss model.

3.1. The Rule of Mixtures: Numerical Example

(from Kollár and Springer 2003)

Consider a Graphite/Epoxy unidirectional ply. Matrix properties are given with subscript m in the table below. Nominal properties with fiber volume fraction $v_f = 60\,\%$ are also given. Assume that the fibers show anisotropy ($E_{f1} \neq E_{f2}$).

	E_1	E_2	G_{12}	ν_{12}	E_m	G_m	ν_m
Value	148	9.65	4.55	0.3	4.1	1.5	0.35

All modulii in GPa.

Estimate the following:

- Fiber modulus properties
- Composite material modulii for volume fraction $v_f = 0.55$.

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