AS2070: Aerospace Structural Mechanics

Module 2: Composite Materials

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Dept. of Aerospace Engg., IIT Madras, Chennai

January 22, 2025

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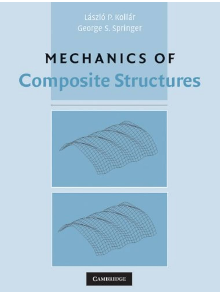
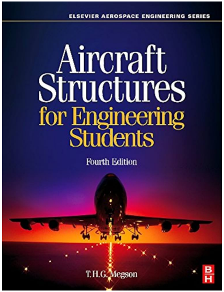


Chapters 1-3 in Kollár and Springer (2003).

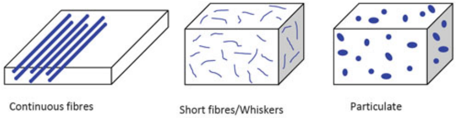


Chapter 25 in Megson (2013)

1.1. What are Composites?

Introduction

- Structural material consisting of multiple non-soluble macro-constituents.
- Main motivation: material properties tailored to applications.
- Both stiffness and strength comes from the fibers/particles, and the matrix holdes everything together.



Types of composite materials (*Figure from NPTEL Online-IIT KANPUR (n.d.)*)

Examples

- Reinforced concrete
- Wood (lignin matrix reinforced by cellulose fibers)
- Carbon-Fiber Reinforced Plastics (CFRP)

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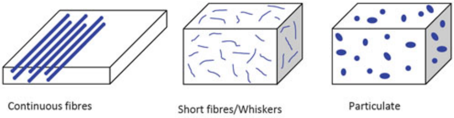

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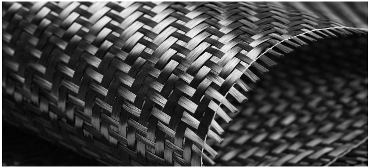
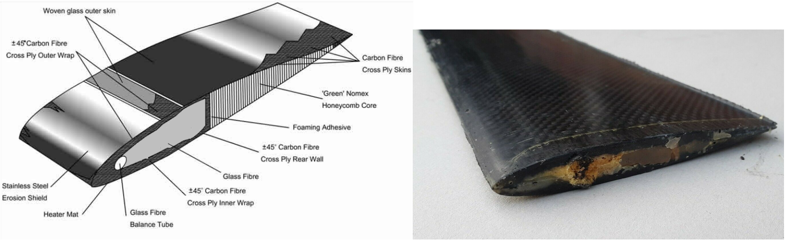


1.1. What are Composites?

Introduction

- Structural material consisting of multiple non-soluble macro-constituents.

CFRP Helicopter Blades



(Figures from Carbon Fiber Top Helicopter Blades n.d.)

Ex

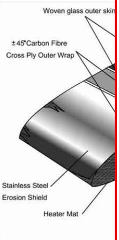
- Reinforced concrete
- Wood (lignin matrix reinforced by cellulose fibers)
- Carbon-Fiber Reinforced Plastics (CFRP)

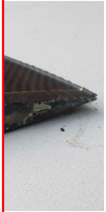
- ~2x stiffness, ~3x strength, ~ 70% weight of AA.
- High fatigue resistance. But quite brittle.
- Main- and tail-planes, fuselages, etc. Helicopter blades.

1.1. What are Composites?

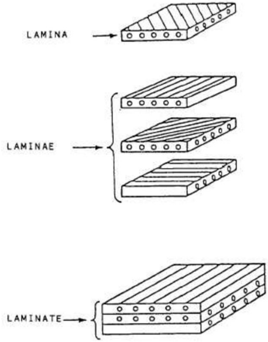
Introduction

- Structural material consisting of multiple non-soluble macro-constituents.
- M





Laminated Composites



(*Figure from Kalkan 2017*)

Ex

- Reinforced co
- Wood (lignin cellulose fiber
- Carbon-Fiber Reinforced Plastics (CFRP)

gth, ∼ 70%

But quite brittle.

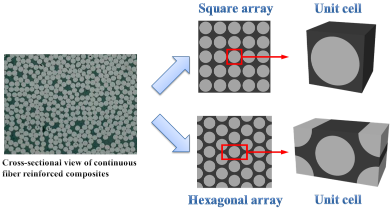
- Main- and tail-planes, fuselages, etc. Helicopter blades.

1.2. Modeling Composite Material

Introduction

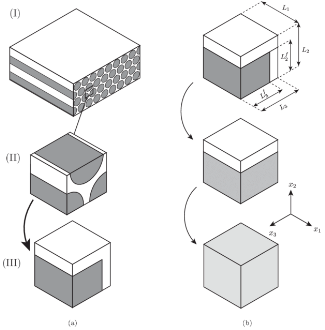
Two main approaches:

Micro-Mechanics



(Figure from "Micro-Mechanics of Failure" 2024)

Macro-Mechanics



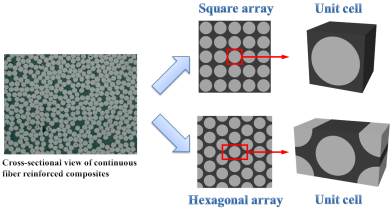
Homogenization of micro-structure (Figure from Skovsgaard and Heide-Jørgensen 2021)

1.2. Modeling Composite Material

Introduction

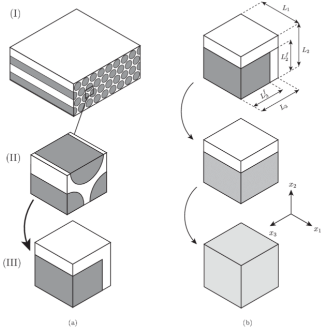
Two main approaches:

Micro-Mechanics



(Figure from "Micro-Mechanics of Failure" 2024)

Macro-Mechanics ✓

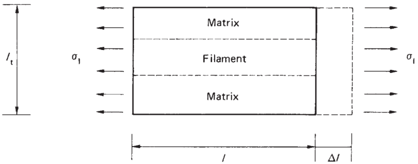


Homogenization of micro-structure (Figure from Skovsgaard and Heide-Jørgensen 2021)

1.3. Constitutive Modeling for Composites

Introduction

Axial Elongation



- Strain is fixed, but stress experienced by media differ.

$$\sigma_l = E_l \varepsilon_l$$

- Stress-strain relationship simplifies as,

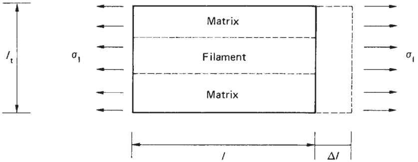
$$\sigma_m = E_m \varepsilon_l, \quad \sigma_f = E_f \varepsilon_l$$
$$\sigma_l A = \sigma_m A_m + \sigma_f A_f$$
$$\implies \boxed{E_l = \frac{A_f}{A} E_f + \frac{A_m}{A} E_m}.$$

(Figures from Megson 2013)

1.3. Constitutive Modeling for Composites

Introduction

Axial Elongation



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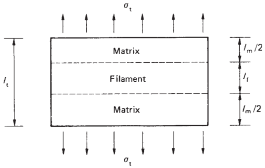
Transverse Elongation



- Stress is fixed, strains differ:

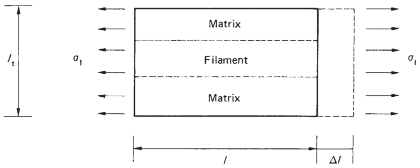
$$\varepsilon_t l_t = \varepsilon_m l_m + \varepsilon_f l_f$$
$$\implies \frac{\sigma_t}{E_t} l_t = \frac{\sigma_t}{E_m} l_m + \frac{\sigma_t}{E_f} l_f$$
$$\implies \boxed{\frac{1}{E_t} = \frac{1}{E_m}\frac{l_m}{l_t} + \frac{1}{E_f}\frac{l_f}{l_t}}.$$

(Figures from Megson 2013)

1.3. Constitutive Modeling for Composites

Introduction: Poisson Effects

Axial-Transverse Coupling



- Transverse displacement written as

$$\Delta_t = \nu_m \varepsilon_l l_m + \nu_f \varepsilon_l l_f := \nu_{lt} \varepsilon_l l_t$$

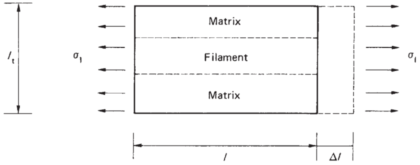
$$\implies \boxed{\nu_{lt} = \frac{l_m}{l_t}\varepsilon_l + \frac{l_f}{l_t}\varepsilon_f}.$$

(Figures from Megson 2013)

1.3. Constitutive Modeling for Composites

Introduction: Poisson Effects

Axial-Transverse Coupling



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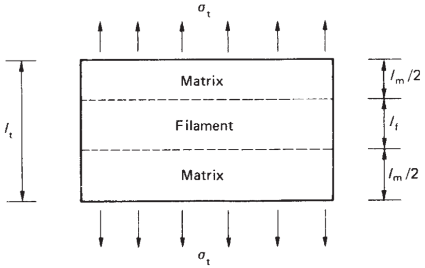
Transverse-Axial Coupling



- Axial displacement written as

$$\nu_m \frac{\sigma_t}{E_m} = \nu_f \frac{\sigma_t}{E_f} := \nu_{tl} \frac{\sigma_t}{E_t},$$

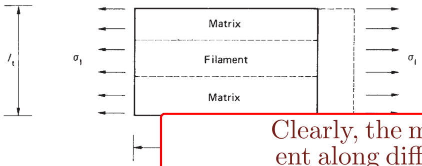
$$\implies \boxed{\nu_{tl} = \frac{E_t}{E_l}\nu_{lt}}.$$

(Figures from Megson 2013)

1.3. Constitutive Modeling for Composites

Introduction: Poisson Effects

Axial-Transverse Coupling



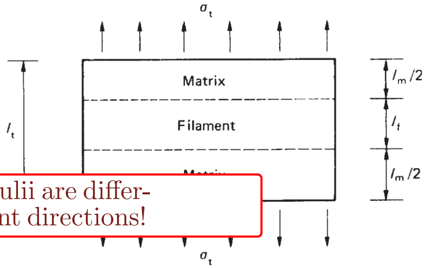
Clearly, the modulii are different along different directions!

- Transverse displacement written as

$$\Delta_t = \nu_m \varepsilon_l l_m + \nu_f \varepsilon_l l_f := \nu_{lt} \varepsilon_l l_t$$

$$\implies \boxed{\nu_{lt} = \frac{l_m}{l_t}\varepsilon_l + \frac{l_f}{l_t}\varepsilon_f}.$$

Transverse-Axial Coupling



- Axial displacement written as

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$$\implies \boxed{\nu_{tl} = \frac{E_t}{E_l}\nu_{lt}}.$$

(Figures from Megson 2013)

1.3. Constitutive Modeling for Composites

Introduction: Anisotropy

General Anisotropy

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16}\\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26}\\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36}\\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46}\\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56}\\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66}\end{bmatrix} \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix}$$

1.3. Constitutive Modeling for Composites

Introduction: Anisotropy

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Monoclinic: Single Plane of Symmetry

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & 0 & 0\\ C_{12} & C_{22} & C_{23} & C_{24} & 0 & 0\\ C_{13} & C_{23} & C_{33} & C_{34} & 0 & 0\\ C_{14} & C_{24} & C_{34} & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & C_{56}\\ 0 & 0 & 0 & 0 & C_{56} & C_{66}\end{bmatrix} \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix}$$

1.3. Constitutive Modeling for Composites

Introduction: Anisotropy

Triclinic: Three Planes of Symmetry

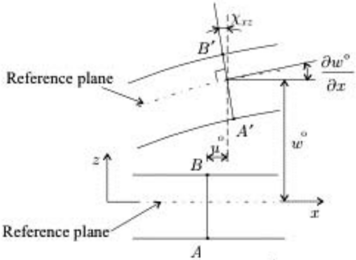
$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{bmatrix} = \begin{bmatrix}C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix} \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix}$$

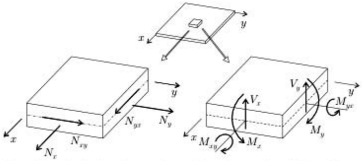
Transversely Isotropic

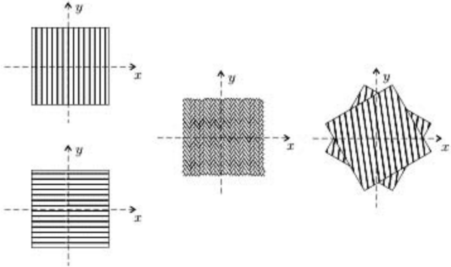
$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{bmatrix} = \begin{bmatrix}C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{13} & 0 & 0 & 0\\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2}\end{bmatrix} \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix}$$

1.4. Classical Laminate Theory

Introduction







Figures from Kollár and Springer 2003

References I

[1] L. P. Kollár and G. S. Springer. Mechanics of Composite Structures, Cambridge: Cambridge University Press, 2003. ISBN: 978-0-521-80165-2. DOI: 10.1017/CBO9780511547140. URL: https://www.cambridge.org/core/books/mechanics-of-composite-structures/804A0A5EE67784D7172E142559979445 (visited on 01/11/2025) (cit. on pp. 2, 17).

[2] T. H. G. Megson. Aircraft Structures for Engineering Students, Elsevier, 2013. ISBN: 978-0-08-096905-3 (cit. on pp. 2, 9–13).

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[5] Ş. Kalkan. "TECHNICAL INVESTGATION FOR THE USE OF TEXTILE WASTE FIBER TYPES IN NEW GENERATION COMPOSITE PLASTERS". PhD thesis. July 2017 (cit. on pp. 3–6).

[6] "Micro-Mechanics of Failure". Wikipedia, (May 2024). URL: https://en.wikipedia.org/w/index.php?title=Micro-mechanics_of_failure&oldid=1224426361 (visited on 01/22/2025) (cit. on pp. 7, 8).

[7] S. Skovsgaard and S. Heide-Jørgensen. "Three-Dimensional Mechanical Behavior of Composite with Fibre-Matrix Delamination through Homogenization of Micro-Structure". Composite Structures, **275**, (July 2021), pp. 114418. DOI: 10.1016/j.compstruct.2021.114418 (cit. on pp. 7, 8).