AS2070: Aerospace Structural Mechanics

Module 1: Elastic Stability

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Dept. of Aerospace Engg., IIT Madras, Chennai

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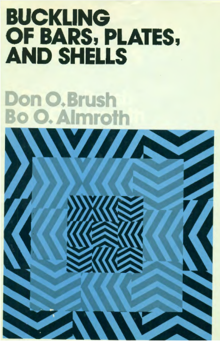
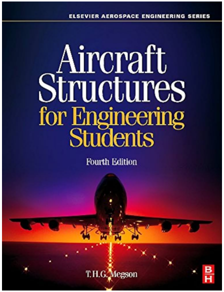


Chapters 1-3 in Brush and Almroth (1975).

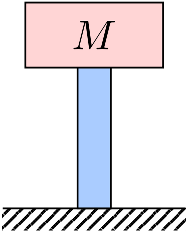


Chapters 7-9 in Megson (2013)

1. Introduction

Structural Stability: What?

- Consider supporting a mass M on the top of a rod.



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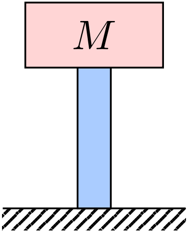
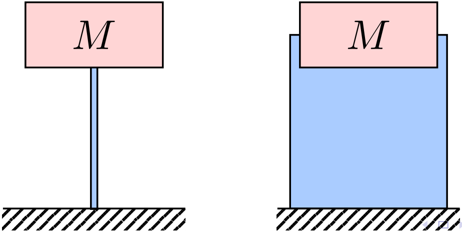


Two Extreme Cases:



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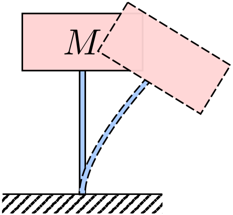

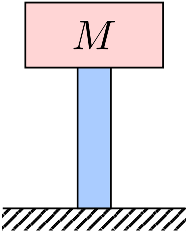

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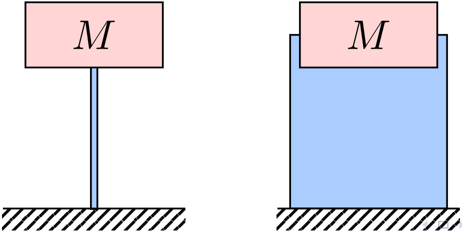

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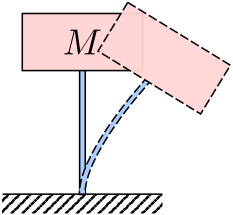
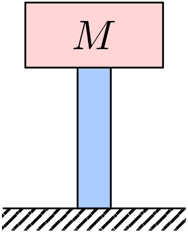


How can we mathematically describe this?



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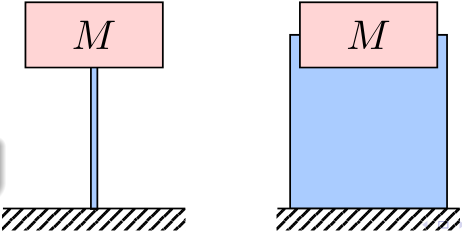


1. Introduction

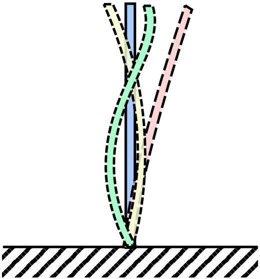
Structural Stability: Perturbation Behavior

Perturbation Behavior

Key insight we will invoke is behavior under **perturbation**:

How would the system respond if I slightly perturb it?

- Mathematically, by perturbation we mean *any change to the system's configuration.*
- In this case, this could be different deflection shapes.



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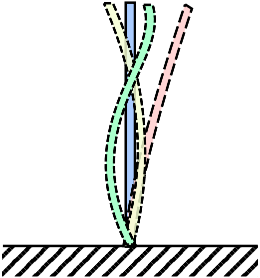


Question (Slightly more specific)

What will the system tend to do if an arbitrarily small magnitude of perturbation is introduced?

- Will it tend to **return to its original configuration**?
- Will it **blow up**?
- Will it do **something else entirely**?

1.1. Elastic Stability

Introduction

What do these words mean?

Elastic → Reversible → Conservative

Conservative System

- The restoring force of a conservative system can be written using a gradient of a **potential function**:

$$\underline{F} = -\nabla U.$$

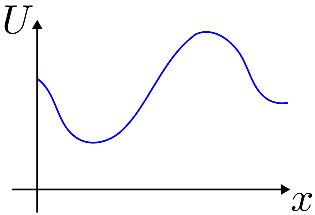
Equilibrium

- System achieves equilibrium when $\underline{F} = \underline{0}$, i.e.,

$$\nabla U = 0.$$

1D Example

Consider a system whose configuration is expressed by the scalar x and the potential is as shown.



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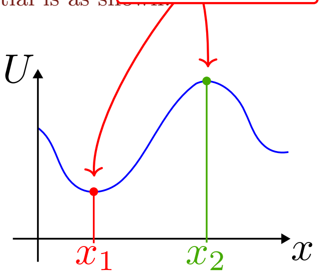

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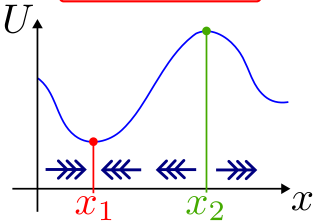
Remember, $F = -\frac{dU}{dx}$.



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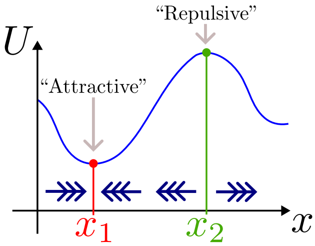

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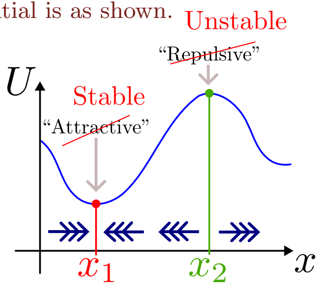
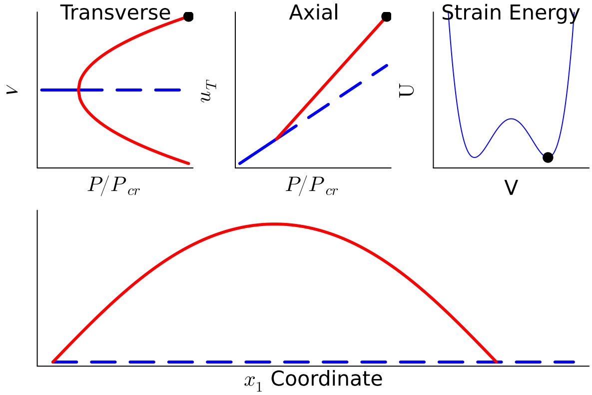


1.2. Bifurcation

Introduction

A system is said to have **undergone a bifurcation** if its state of stability has changed due to the variation of some parameter.

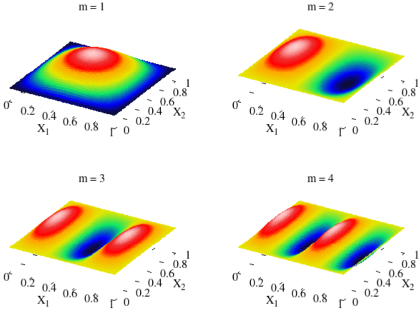


Example: *A pinned-pinned beam undergoing axial loading.*

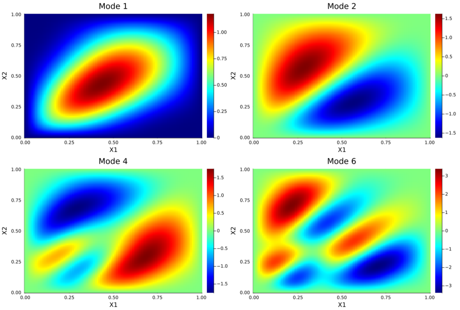
1.3. Modes of Stability Loss

Introduction

The **configuration** that a system can assume as it undergoes a bifurcation is the *mode* of the stability loss.



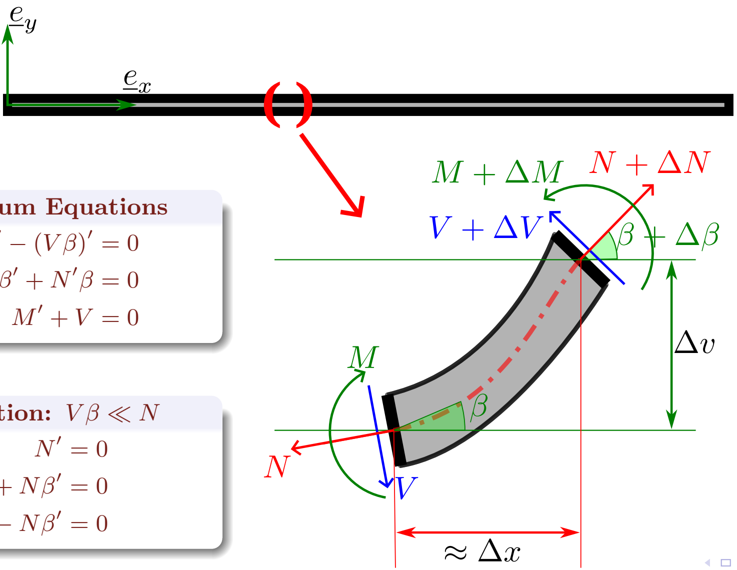
Example: *Thin plate (pinned) under axial loading*



Example: *Thin plate (pinned) under shear loading*

2.1. Equilibrium Equations

Euler Buckling of Columns



Equilibrium Equations

$$N' - (V\beta)' = 0$$
$$V' + N\beta' + N'\beta = 0$$
$$M' + V = 0$$

Assumption: $V\beta \ll N$

$$N' = 0$$
$$V' + N\beta' = 0$$
$$M'' - N\beta' = 0$$

2.2. Kinematic Description

Euler Buckling of Columns

References I

[1] D. O. Brush and B. O. Almroth. Buckling of Bars, Plates, and Shells, McGraw-Hill, 1975. ISBN: 978-0-07-008593-0 (cit. on p. 2).

[2] T. H. G. Megson. Aircraft Structures for Engineering Students, Elsevier, 2013. ISBN: 978-0-08-096905-3 (cit. on p. 2).

4. Class Discussions (Outside of Slides)

- Ball on a hill. 2D, 3D cases.
- Assumptions behind compression of a bar.