AS 203 Gas Dynamics

Solutions to Practice Problems - 1

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NOTE: Specific gas contant $R = 287$ J/Kg/K and $\gamma = 1.4$ is used wherever not mentioned.

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1. Gas at $P_o = 10$ MPa and $T_o = 1000K$ is expanded to $p = 1$ atm $= 1.01325 * 10^5 Pa$
Assuming adiabatic expansion:

$$\frac{P_o}{p} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

Substituting p, Po and $\gamma = 1.4$ we get $M = 3.68$
Also,

$$\frac{T_o}{T} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right)$$

So, at $T_o = 1000K$ and $M = 3.68$ we get $T = 269.7K$

$$M = \frac{v}{c} = \frac{v}{\sqrt{\gamma RT}}$$

Substituting values we get $v = 1211.42$ m/s

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2. Air moving at $v = 1000$ m/s at $p = 1atm = 1.01325 * 10^5 Pa$ and $T = 300K$ is isentropically expanded to $T = 100K$

$$M_1 = \frac{v}{\sqrt{\gamma RT}} = 2.88$$

From gastables for isentropic flows at $M_1 = 2.88$ we get $T/T_o = 0.3761$ and $p/P_o = 0.0326$.
Using the given static temperature and pressure values we get: $T_o = 797.67K$ and $P_o = 3.11MPa$

Since T_o reamins contant due to isentropic expansion, at the final temperature $T = 100K$, we have $T/T_o = 0.1254$. Using gastbales we get $M_2 = 5.90$ and $p_2/P_o = 0.0007$.

$$v_2 = M_2\sqrt{\gamma RT} = 1182.65 m/s$$

and

$$p_2 = 21756.9 Pa$$

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3. Special gas ($\gamma = 1.3$ and $MW = 18$), therefore

$$R_{gas} = \frac{8314.3}{18} = 461.90 \qquad J/Kg/K$$

Argon ($\gamma = 1.67$ and $MW = 40$), therefore

$$R_{argon} = \frac{8314.3}{40} = 207.858 \qquad J/Kg/K$$

Since the Mach numbers are same

$$\left(\frac{v}{\sqrt{\gamma RT}}\right)_{gas} = \left(\frac{v}{\sqrt{\gamma RT}}\right)_{argon}$$

Therefore,

$$\frac{v_{gas}}{v_{argon}} = \frac{(\sqrt{\gamma RT})_{argon}}{(\sqrt{\gamma RT})_{gas}} \tag{1}$$

Since,

$$\frac{T_o}{T} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right)$$

and stagnation conditions are same we can write:

$$\frac{T_{argon}}{T_{gas}} = \frac{(1 + \frac{(\gamma-1)}{2}M^2)_{gas}}{(1 + \frac{(\gamma-1)}{2}M^2)_{argon}} \tag{2}$$

Using (1) and (2) we get:

$$\frac{v_{gas}}{v_{argon}} = 1.84775$$

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4. Supersonic aircraft is flying at $h = 5km$

$$T = T_o - Lh$$

and

$$p = P_o \left(\frac{T_h}{T_o}\right)^{g/RL}$$

where L (Lapse rate) = 0.0065 K/m, T_o (Sea level) = 288 K and P_o (Sea level) = 1.01325 * 10^5 Pa

So, at h = 5 km = 5000 m we get T = 255.5 K and p = 0.55 * 10^5 Pa
The maximum pressure that the structure can withstand is p = 10 bar = 10 * 10^5 Pa

Considering body fixed co-ordinates we need to find the maximum permissible mach number for p_1 = 0.55 * 10^5 Pa and $P_{o_2} = 10^6$ Pa

Using gastables for normal shock relations, for

$$\frac{P_{o_2}}{p_1} = 18.18$$

we get mach number M = 3.70

Therefore, the maximum relative airspeed will be

$$v = M * \sqrt{\gamma RT_1} = 1185.5m/s$$

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5. A meteor is flying at M=25. Using normal shock relations at M = 25 we get:

$$\frac{T_2}{T_1} = 122.47$$

Now, maximum T_2 that the structure can withstand is 1300 K, so maximum permissible T_1 is

$$T_1 = 10.61K$$

The altitude corresponding to this temperature will be somewhere in Mesosphere 60-70 Km

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6. Shock velocity V_s = 2000 m/s. T_o = 500 K and P_o = 3 atm = 3 * 1.01325 * 10^5 Pa

Using shock fixed co-ordinates:

$$M_1 = \frac{v}{\sqrt{\gamma RT}} = 4.46$$

Using gastables for Normal Shock at M_1 = 4.46 we get M_2= 0.4243, p_2/p_1 = 23.0402 and T_2/T_1 = 4.8053
In shock fixed co-ordinates:

$$T_1 = T_o = 500K \qquad and \qquad p_1 = P_o = 3.03975 * 10^5 Pa$$

Therefore,

$$T_2 = 2402.65K \qquad p_2 = 70.0364 * 10^5 Pa$$

Let the actual velocity behind the shock in shock free co-ordinates be u_2. Mach number behind the shock in shock fixed co-ordinates is given as:

$$M_2 = \frac{V_s - u_2}{\sqrt{\gamma R T_2}}$$

Therefore, actual Mach number behind the shock in shock free co-ordinates will be:

$$M_{2_{free}} = \frac{u_2}{\sqrt{\gamma R T_2}} = \frac{V_s}{\sqrt{\gamma R T_2}} - M_2$$

$$M_{2_{free}} = \frac{u_2}{\sqrt{\gamma R T_2}} = \frac{V_s}{\sqrt{\gamma R T_2}} - M_2 = 2.03554 - 0.4243 = 1.61124$$

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7. Shock velocity V_s = -800 m/s and flow velocity is 50 m/s. Therefore in shock free co-ordinates v_1 = 50 m/s Also, T_1 = 300 K and p_1 = 1 atm = 1.01325 * 10^5 Pa

In shock fixed co-ordinates:

$$M_1 = \frac{v_1 - V_s}{\sqrt{\gamma R T}} = 2.45$$

Using gastables for Normal Shock at $M_1 = 2.45$ we get $M_2 = 0.5179$, $p_2/p_1 = 6.8363$ and $T_2/T_1 = 2.0885$

Using the given values we get:

$$T_2 = 626.55K \qquad p_2 = 6.92688 * 10^5 Pa$$

Let the actual velocity behind the shock in shock free co-ordinates be u_2. Mach number behind the shock in shock fixed co-ordinates is given as:

$$M_2 = \frac{V_s - u_2}{\sqrt{\gamma R T_2}}$$

Therefore, actual Mach number behind the shock in shock free co-ordinates will be:

$$M_{2_{free}} = \frac{u_2}{\sqrt{\gamma R T_2}} = \frac{V_s}{\sqrt{\gamma R T_2}} - M_2$$

$$M_{2_{free}} = \frac{u_2}{\sqrt{\gamma R T_2}} = \frac{V_s}{\sqrt{\gamma R T_2}} - M_2 = 1.5944 - 0.5179 = 1.0765$$

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8. Similar to problem 6, we have shock moving with velocity V_s = 800 m/s, T_1 = 300 K and p_1 = 1 atm = 1.01325 * 10^5 Pa
In shock fixed co-ordinates:

$$M_1 = \frac{v_1 - V_s}{\sqrt{\gamma R T}} = 2.304$$

Using gastables for Normal Shock at $M_1 = 2.304$ we get $M_2 = 0.5339$, $p_2/p_1 = 6.0265$ and $T_2/T_1 = 1.9505$

Using the given values we get:

$$T_2 = 585.15K \qquad p_2 = 6.10635 * 10^5 Pa$$

Now, using gastables for isentropic flow at $M_2 = 0.5339$ we get $p/P_o = 0.8236$
So, the maximum pressure experienced by the building is:

$$P_o = p_2/0.8236 = 7.41422 * 10^5 Pa = 7.317 atm$$

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9. Using gastables for weak oblique shock at $M_1 = 3$ and $\theta = 15$ we get for,

- $\gamma = 1.3$: wave angle = 31.53, $p_2/p_1 = 2.7048$ and $T_2/T_1 = 1.3666$
- $\gamma = 1.4$: wave angle = 32.2404, $p_2/p_1 = 2.8216$ and $T_2/T_1 = 1.3883$
- $\gamma = 1.67$: wave angle = 34.2851, $p_2/p_1 = 3.1652$ and $T_2/T_1 = 1.4511$

10. $M_1 = 3$, $p_1 = 2$ atm and $T = 250$ K.
Using gastables for isentropic flow ta $M_1 = 3$ we get: $p_1/P_{o_1} = 0.0272$ and $T_1/T_{o_1} = 0.3571$ So,

$$P_{o_1} = 73.529atm \qquad and \qquad T_{o_1} = 700.084K$$

From gastables for Oblique shock at $M = 3$ and $\theta = 5$ we get: $M_2 = 2.7497$, $p_2/p_1 = 1.454$, $T_2/T_1 = 1.1146$ and $P_{o_2}/P_{o_1} = 0.9947$

$$P_{o_2} = 73.139atm \qquad T_2 = 278.65K \qquad p_2 = 2.908atm$$

At the top the flow will again deflect by $\theta = 5$ degrees to become parallel with the wall. Using gastables for Oblique shocks at $M_2 = 2.7497$ and $\theta = 5$ we get: $M_3 = 2.5216$, $p_3/p_2 = 1.416$, $T_3/T_2 = 1.1058$ and P_{o_3}/P_{o_2} $= 0.9957$

$$P_{o_3} = 72.824atm \qquad T_3 = 308.131K \qquad p_3 = 4.117atm$$